UNITUE-THEP-17/93
hep-ph/9402236

HEAVY–LIGHT MESONS IN A BILOCAL EFFECTIVE THEORY

Yu. L. Kalinovsky[ab]

MPG AG "Theoretische Vielteilchenphysik"
Universität Rostock
Universitätsplatz 1, D-18051 Rostock, Germany

C. Weiss[cd]

Institut f. Theoretische Physik
Universität Tübingen
Auf der Morgenstelle 14, D-72076 Tübingen, Germany

Abstract

Heavy–light mesons are described in an effective quark theory with a two–body vector–type interaction. The bilocal interaction is taken to be instantaneous in the rest frame of the bound state, but formulated covariantly through the use of a boost vector. The chiral symmetry of the light flavor is broken spontaneously at mean field level. The framework for our discussion of bound states is the effective bilocal meson action obtained by bosonization of the quark theory. Mesons are described by 3–dimensional wave functions satisfying Salpeter equations, which exhibit both Goldstone solutions in the chiral limit and heavy–quark symmetry for $m_Q \to \infty$. We present numerical solutions for pseudoscalar D– and B–mesons. Heavy–light meson spectra and decay constants are seen to be sensitive to the description of chiral symmetry breaking (dynamically generated vs. constant quark mass).

[a]Permanent address: Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Head Post Office, P.O. Box 79, 101000 Moscow, Russia

[b]e-mail address: yura@darss.mpg.uni-rostock.de

[c]Supported by a research fellowship of the Deutsche Forschungsgemeinschaft (DFG)

[d]e-mail address: weiss@mailserv.zdv.uni-tuebingen.de

1 Introduction

One of the most interesting problems of hadronic physics is the study of hadrons consisting of heavy and light quarks. For example, B–meson decays play an important role in determining the elements of the Kobayashi–Maskawa matrix, including the CP–violating phase [1]. Furthermore, rare decays of heavy mesons may indicate deviations from the standard model. For the description of physical (hadronic) decay processes one needs to know the meson wave functions and form factors.

Heavy–light systems are also challenging from the point of view of strong interactions. The physics of light flavors is largely governed by the chiral symmetry of QCD and its spontaneous breaking in the vacuum. In particular, this entails the existence of light pseudoscalar Goldstone bosons, the pions. A simple model exemplifying the dynamical mechanism of chiral symmetry breaking is the Nambu–Jona-Lasinio model, which is based on a local two–body interaction between quarks [2, 3, 4]. For heavy flavors (c, b) the situation is quite different. In this realm chiral symmetry breaking plays a minor role. The strong dynamics of heavy quarks simplifies because of the fact that they behave essentially like classical particles, *i.e.*, their off–shellness in a bound state is small compared to the quark mass. Thus, heavy quarkonia $(c\bar{c}, b\bar{b})$ are successfully described by non-relativistic potential models using a Coulomb–type interaction at short distances and a linear confinement potential at large distances [5, 6]. Heavy–light hadrons occupy an intermediate position. In heavy–light bound states, the velocity of the heavy quark is conserved in the limit $m_Q \to \infty$, and the mass spectrum becomes independent of the heavy–quark spin. Recently, a general effective theory for heavy–light mesons has been constructed on the basis of both the chiral symmetry of the light flavors and the heavy–quark limit for the heavy flavors [7, 8]. However, the coefficients of this effective Lagrangian are not determined by the symmetries, but by the details of the underlying dynamics, QCD. Thus, for a quantitative understanding of heavy–light systems it is necessary to consider a suitable approximation to QCD at quark level. This approximation must take into account certain qualitative features of QCD, most importantly quark confinement and the spontaneous breaking of chiral symmetry. Moreover, it should incorporate the Coulomb interaction between quarks at short distances. These requirements make it necessary to consider models with non-local effective interactions, which can simulate both short–distance and non-perturbative long–distance effects.

Here we consider the possibility of describing heavy–light mesons in the framework of a bilocal effective quark theory [9, 10]. To incorporate the spontaneous breaking of chiral symmetry, we use a Lorentz–vector effective interaction motivated by QCD. A special feature of our approach is that we take the interaction to be instantaneous in the rest frame of the meson bound state [11, 12, 13]. There are a number of reasons for this choice. Such an interaction leads to a transparent description of mesons in terms of Salpeter wave functions, which satisfy simple Schrödinger–type equations. Nevertheless, the relativistic kinematics as well as effects of spontaneous chiral symmetry breaking are incorporated exactly in the form of Foldy–Wouthuysen factors arising in the

reduction of the Bethe–Salpeter equation. Such a model reduces in the non-relativistic limit to the potential model for heavy quarkonia ($Q\bar{Q}$). Furthermore, instantaneous interactions in the form of the Coulomb gauge have long been used to study chiral symmetry breaking in a quasiparticle picture of the QCD vacuum [14, 15, 16, 17].

In general, potential models do not possess full relativistic invariance because the Fock space is restricted to $q\bar{q}$–pairs and an instantaneous interaction is assumed. Nevertheless, in our model the interaction is written in a relativistically covariant form through the use of a boost vector proportional to the bound–state total momentum. Physically, this corresponds to the intuitive picture of a potential moving together with the bound state, as is used to describe moving atoms in QED. Moreover, for heavy–light bound states this boost vector coincides with the 4–velocity of the heavy quark, which provides a natural connection of this approach to heavy–quark effective theory. This relativistic formulation allows us to define Bethe–Salpeter amplitudes and wave functions for moving particles, which is crucial for the calculation of formfactors or decay matrix elements leading to the Isgur–Wise functions. A description of heavy meson and baryon weak decays in the heavy quark limit based on Bethe–Salpeter wave functions has been developed by Hussain *et al.* [18]. Recently, Dai *et al.* have considered a covariant instantaneous interaction in this context [19]. In contrast to these approaches we do not take the heavy–quark limit from the start but formulate the description of bound states for arbitrary quark masses, discussing the simplifications arising in the heavy quark limit afterwards. An advantage of this covariant approach is precisely the possibility to describe in a unified manner both light and heavy mesons, using appropriate potentials. For example, for light quarks one may employ a separable approximation to an intermediate–range potential in momentum space. In this case one obtains a consistently regularized version of the Nambu–Jona–Lasinio model, in which the 3–dimensional momentum space cutoff moves along with the bound state [20]. We should point out that the relativistic formulation using the boost vector is useful also in the light quark sector, *e.g.* in defining the pion decay constant [21].

Nowak *et al.* have established the general relation between dynamics at quark level and heavy quark effective theory by performing a gradient expansion of the fermion determinant in the heavy quark limit [22]. The bilocal meson action derived in our approach may serve as an explicit realization of their ideas. However, we shall consider here the full momentum–dependent theory and do not restrict the effective action to the long–wavelength limit.

In this paper, we first develop the formal framework for the description of meson bound states in a bilocal effective model with a covariant instantaneous interaction, which takes into account the spontaneous breaking of chiral symmetry. We then apply this model to the study of light–light and heavy–light pseudoscalar mesons. Specifically, we want to demonstrate that the momentum dependence of the constituent quark mass, which is a consequence of the dynamical nature of chiral symmetry breaking, has important effects on the spectrum and decay constants of heavy–light mesons. The description of meson transitions and form factors in this unified approach will be given in the future [23].

This paper is organized as follows. In sect. 1 we formulate the effective quark theory and introduce the instantaneous interaction in the covariant formulation [11, 12]. We discuss the phenomenological 3–dimensional potentials considered in the applications. We then bosonize the model and obtain an effective bilocal meson theory. The Schwinger–Dyson equation for the vacuum and the Bethe–Salpeter equation for the meson fluctuations are derived from the effective meson action. We outline the calculation of matrix elements between bound states needed to describe meson decay constants or semileptonic decays. In these derivations we assume a general covariant interaction kernel. In sect. 2 we then discuss in detail the equations describing bound states for the case of a covariant instantaneous interaction. The Schwinger–Dyson equation describes the quark quasiparticle spectrum. The Bethe–Salpeter equation for mesons is rewritten in terms of a Salpeter wave function. We make use of a Foldy–Wouthuysen transformation to simplify the form of the resulting equations. In sect. 3, we then apply the model to the description of light–light and heavy–light mesons. We discuss the chiral limit, $m_q \rightarrow 0$, and study the behavior of the pseudoscalar meson in this regime. We then investigate heavy–light bound states $(Q\bar{q})$. In the heavy quark limit, $m_Q \rightarrow \infty$, reduced bound state equations are obtained, which exhibit the heavy–quark spin symmetry. We present numerical solutions of the full equations for pseudoscalar D– and B–mesons. Specifically, we compare meson properties calculated with the momentum–dependent quark mass from the Schwinger–Dyson–equation with those obtained using a constant light quark mass. We find that heavy–light meson properties are sensitive to whether chiral symmetry breaking for the light flavor is described self–consistently or by a constant constituent quark mass. A summary and an outlook are given in sect. 4.

Appendix A.1 deals with the partial–wave analysis of the meson Salpeter equation. In appendix A.2 we describe the numerical solution of the partial–wave equations using the Multhopp method, which has been very sucessful in the context of constituent quark models [24]. In particular, we consider it necessary to comment on the complications presented by the momentum–dependent Foldy–Wouthuysen factors in the bound state equations of our model. Appendix B outlines the evaluation of the matrix elements for the normalization of meson wave functions and the calculation of meson decay constants from the bilocal effective meson action.

2 Effective quark theory with instantaneous interaction

The basis of our description is a quark theory with an effective two–body interaction in the color–octet channel. It is defined by the action of the quark field,

$$W = \int d^4x \, \bar{q}(x) G_0^{-1}(x) q(x) - \tfrac{1}{2} g^2 \int\int d^4x \, d^4y \; j^{\mu a}(x) \hat{D}^{ab}_{\mu\nu}(x-y) j^{\nu b}(y). \tag{1}$$

Here, G_0 is the free quark Green function,

$$G_0^{-1} \;\; = \;\; i\partial\!\!\!/ - \hat{m}^0, \tag{2}$$

where $\hat{m}^0 = \mathrm{diag}(m_1^0, \ldots, m_{N_f}^0)$ is the quark current mass matrix, with N_f the number of flavors. The quark color current is given as

$$j_\mu^a(x) \;\; = \;\; \bar{q}(x)\Big(\frac{\lambda^a}{2}\Big)\gamma_\mu q(x) \;, \tag{3}$$

where λ^a are the Gell–Mann matrices of $SU(3)_\mathrm{c}$. The bilocal interaction kernel,

$$\hat{D}_{\mu\nu}^{ab}(x-y) \;\; \equiv \;\; \delta^{ab} g_{\mu\nu} D(x-y), \tag{4}$$

can be thought of as an effective gluon propagator, which describes part of the non-abelian effects of QCD in a phenomenological way. Note that the interaction is of Lorentz–vector type, as motivated by QCD, and thus chirally invariant.

For the purpose of describing meson bound states we rewrite the interaction term of eq.(1) in the form

$$\int\!\!\int d^4x\, d^4y\; q_B(y)\bar{q}_A(x)\hat{K}_{AB,CD}(x-y)q_D(x)\bar{q}_C(y), \tag{5}$$

with the kernel

$$\hat{K}_{AB,CD}(x-y) \;\; = \;\; \gamma_{ru}^\mu(\gamma_\mu)_{ts} \sum_{a=1}^{8} \frac{\lambda_{\alpha\delta}^a}{2}\frac{\lambda_{\gamma\beta}^a}{2}\,\delta_{il}\delta_{kj}\;\frac{g^2}{2}D(x-y). \tag{6}$$

Here, $A = \{r, \alpha, i\}, \ldots, D = \{u, \delta, l\}$ are a short–hand notation for the Dirac spinor, color and and flavor indices. In the following we want to study meson $(q\bar{q})$ bound states. We therefore make the well–known color Fierz rearrangement [25]

$$\sum_{a=1}^{8} \lambda_{\alpha\delta}^a \lambda_{\gamma\beta}^a \;\; = \;\; \frac{4}{3}\delta_{\alpha\beta}\delta_{\gamma\delta} \; + \; \frac{2}{3}\sum_{\rho=1}^{3} \epsilon_{\rho\alpha\gamma}\epsilon^{\rho\beta\delta}, \tag{7}$$

where $\epsilon_{\alpha\beta\gamma}$ is the antisymmetric Levi–Civita tensor. This identity allows to rewrite the interaction completely into the attractive color–singlet $(q\bar{q})$ and antitriplet (qq) channels, while the repulsive color–octet and sextet channels are absent in a natural way. We consider only the color–singlet part of eq.(6). The relevant part of the action, eq.(1), can then be represented in the form

$$\begin{aligned} W \;\; = \;\; & \int\!\!\int d^4x\, d^4y\, \Big\{ (q(y)\bar{q}(x))\,(-G_0^{-1}(x))\delta(x-y) \\ & + \frac{1}{2N_c}\Big[(q(y)\bar{q}(x))\, K(x-y)\, (q(x)\bar{q}(y)) \Big] \Big\}, \end{aligned} \tag{8}$$

with the color–singlet interaction kernel

$$K(x-y) \;\; = \;\; \gamma^\mu \otimes \gamma_\mu\, g^2 D(x-y). \tag{9}$$

Here, the bilinear $(q(y)\bar{q}(x))$ is contracted over color indices, and $N_c = 3$ is the number of colors.

If the model action eq.(1) is considered as a Euclidean field theory with a covariant gluon propagator, eq.(1) is known as the so-called Global Color Model [26]. In that approach, bound states are studied through the Euclidean correlation functions. Similar covariant models have been investigated in [27, 28]. Here, we consider eq.(1) in a different context. For reasons already stated above we wish to have an effective theory with an instantaneous interaction. Such an interaction leads to a simple description of bound states in terms of 3–dimensional wave functions in Minkowski space, which for heavy-heavy bound states naturally reduces to the successful non-relativistic potential model. The concept of an instantaneous interaction can be formulated covariantly by letting the potential move along with the bound state.

Consider a bound state of two quarks interacting through a bilocal effective interaction of the form eq.(9). From the principle of translational invariance it follows that one can separate the center–of–mass motion of the bound state from the relative motion in the form of a plane wave. The momentum of the c.o.m. motion equals the total momentum of the quark pair, $\mathcal{P}_\mu$. Given this, one can more or less arbitrarily define a conjugate coordinate, X_μ, representing the absolute position in space–time and a relative coordinate, z_μ, to describe the internal structure of the bound state. The condition is that under a translation by a constant vector a, $x \to x+a, y \to y+a$, these coordinates transform as $X \to X + a, z \to z$. This is satisfied for any linear combination $X = \alpha x + (1-\alpha)y$ and $z = x - y$. We shall take $\alpha = \frac{1}{2}$ in the following[1].

We now want to substitute the general bilocal interaction kernel of eq.(9) by an instantaneous one, *i.e.*, by a potential. This can be done in a relativistically covariant way by replacing eq.(9) by

$$K^\eta(x,y) \;=\; K^\eta\left(x-y\,\middle|\,\frac{x+y}{2}\right) \;=\; -\not\eta \otimes \not\eta\, V(z^\perp)\,\delta(z^{||}). \tag{10}$$

Here,

$$z_\mu^{||} \;=\; \eta_\mu (z\cdot\eta), \qquad z_\mu^\perp \;=\; z_\mu - z_\mu^{||}, \tag{11}$$

and η_μ is a boost vector proportional to the total momentum eigenvector of the bound state, $\mathcal{P}_\mu$,

$$\eta_\mu \;=\; \frac{\mathcal{P}_\mu}{\sqrt{\mathcal{P}^2}}, \qquad \eta^2 \;=\; 1, \qquad \not\eta \;=\; \eta_\mu\gamma_\mu. \tag{12}$$

In eq.(10), the δ–function, $\delta(z\cdot\eta)$, guarantees the instantaneousness of the exchange interaction in the rest frame of the bound state. The transversality of the exchange interaction is ensured by the fact that the 3–dimensional potential is a function only

[1] The definition of the center–of–mass coordinate as $X = (m_1 x + m_2 y)/(m_1 + m_2)$ has significance only in the non-relativistic limit. In a relativistic theory it is in general impossible to write the two–body hamiltonian as the sum of two terms describing c.o.m. and relative motion.

of the perpendicular part of the relative coordinate, $V(z^{\perp})$. The sign in eq.(10) has been chosen such that an attractive 3–dimensional potential leads to to an attractive interaction in the color–singlet (and antitriplet) channel. Eq.(10) describes a potential moving together with the bound state. This form of interaction leads to bound–state amplitudes with well–defined Lorentz transformation properties. We remark that one can arrive at the form eq.(10) by studying moving bound states within a canonical quantization approach to gauge theories [29].

For the bound state at rest one has $\vec{\mathcal{P}} = 0$, so that $\eta_\mu = (1, \vec{0})$. In this frame the kernel takes the form

$$K^{(1,\vec{0})}(x,y) \;=\; -\gamma_0 \otimes \gamma_0 V(\mathbf{x}-\mathbf{y})\delta(x_0 - y_0). \tag{13}$$

In electrodynamics this kernel corresponds to the usual Coulomb gauge, with $V(\mathbf{x}-\mathbf{y})$ the Coulomb potential. The form eq.(13) has been widely used as a model to describe the breaking of chiral symmetry in strong interactions [14, 15, 16, 17].

Let us now assume that the bound state contains a heavy quark (c, b) and a light antiquark $(\bar{u}, \bar{d}, \bar{s})$. In the limit of heavy quark effective theory, $m_Q \to \infty$, one has

$$\eta_\mu \;=\; \frac{\mathcal{P}_\mu}{\sqrt{\mathcal{P}^2}} \;\to\; v_\mu, \tag{14}$$

where v is the 4–velocity of the heavy quark, $v^2 = 1$. Thus, in this limit the interaction kernel, eq.(10), takes the form

$$K^v(x-y) \;=\; -\not{v} \otimes \not{v} V(z^{\perp})\delta(v{\cdot}z), \tag{15}$$

where $z^{\perp} = z - v(v{\cdot}z)$. The fact that the interaction kernel explicitly involves the heavy quark 4–velocity leads to a natural relation of this potential approach with heavy–quark effective theory [7, 8].

The transverse potential, $V(z^{\perp})$, is a phenomenological input to this model. Its form may be chosen depending on the type of bound state one wishes to study (light–light, heavy–light). We shall in the following work with a potential, which in the rest frame is of the form

$$V(\mathbf{x}-\mathbf{y}) \;=\; -\frac{4}{3}\frac{\alpha_s}{r} + \sigma^2 r, \tag{16}$$

with $r = |\mathbf{x}-\mathbf{y}|$. Note that when expressing the interaction in an arbitrary frame one must take into account that $z^{\perp}$ is subject to a non-Euclidean metric. The first term of eq.(16) is the Coulomb potential describing one–gluon exchange. In the investigations in this paper we shall take α_s as constant. One could also employ in this approach an asymptotically free potential with $\alpha_s = \alpha_s(|\mathbf{q}|^2)$ [16, 17]. The linear potential in eq.(16) implements quark confinement in a phenomenological way. In our study of heavy–light mesons we use parameters determined within the non-relativistic potential model for charmonium, $\alpha_s \sim 0.3 \ldots 0.5$, $\sigma \sim 0.4\,\mathrm{GeV}$ [5, 6]. For reference, we shall also consider the oscillator model of Le Yaouanc *et al.*, $V(\mathbf{x}-\mathbf{y}) = V_0^3 r^2$, with $V_0 \sim 0.2 \ldots 0.4\,\mathrm{GeV}$,

which gives a good overall fit to the light meson mass spectrum [21]. We remark that one may include in eq.(16) also an intermediate–range attractive potential, which can be thought of as a crude representation of instanton effects [30, 31]. Another possibility is a separable potential in momentum space, which leads to a Nambu–Jona–Lasinio–type model. Such an interaction has been considered within this covariant approach in [20].

3 Bound states

3.1 Hadronization

For the discussion of meson bound states it is convenient to "hadronize" the quark theory, *i.e.*, to formally rewrite it as an effective meson theory [3, 10]. For the general discussion here we assume for simplicity a general covariant interaction kernel. The equations obtained will then be specified to the case of a co-moving instantaneous interaction, eq.(10), in the following two sections.

Let us consider the functional integral

$$Z \;=\; \int \mathcal{D}q\mathcal{D}\bar{q}\; \exp iW[\bar{q}, q]. \tag{17}$$

Here, W is the color–singlet part of the effective quark action, eq.(8), which in symbolic notation can be written as

$$W[\bar{q}, q] \;=\; (q\bar{q}, -G_0^{-1}) \;+\; \frac{1}{2N_c}(q\bar{q}, K\, q\bar{q}). \tag{18}$$

After integrating over the quark fields with the help of the Legendre transform one obtains

$$Z \;=\; \int \mathcal{D}\mathcal{M}\; \exp iW_{\text{eff}}[\mathcal{M}], \tag{19}$$

with the effective meson action

$$W_{\text{eff}}[\mathcal{M}] \;=\; N_c\Big\{ -\tfrac{1}{2}(\mathcal{M}, K^{-1}\mathcal{M}) - i\mathrm{Tr}\,\log(-G_0^{-1} + \mathcal{M})\Big\}. \tag{20}$$

Here, $\mathcal{M} = \mathcal{M}_{ij}(x, y) \sim q_i(x)\bar{q}_j(y)$ is a bilocal meson (color–singlet) field. It has the structure of a matrix in Dirac spinor and flavor space. In eq.(20), the symbol Tr implies integration over the continuous variables as well as the traces over spinor and flavor indices.

The vacuum of the effective meson theory is given as the minimum of the effective action, eq.(20). The condition of minimum reads

$$K^{-1}\mathcal{M} + i\frac{1}{-G_0^{-1} + \mathcal{M}} \;=\; 0. \tag{21}$$

The vacuum solution of this equation is translationally invariant. Let us denote it by $(\Sigma - \hat{m}^0)$. Then we obtain from eq.(21) the Schwinger–Dyson equation

$$\Sigma \;=\; \hat{m}^0 + iKG_\Sigma, \tag{22}$$

where

$$G_\Sigma^{-1} \;=\; i\partial\!\!\!/ - \Sigma. \tag{23}$$

Mesons are described as fluctuations of $\mathcal{M}$ around the vacuum configuration. Expanding the action, eq.(20), around the minimum, with $\mathcal{M} = (\Sigma - \hat{m}^0) + \mathcal{M}'$, one obtains

$$\begin{aligned} W_{\rm eff}[\mathcal{M}] \;&=\; W_{\rm eff}[\Sigma] \\ &+\; N_c\Big\{-\frac{1}{2}(\mathcal{M}', K^{-1}\mathcal{M}') - \frac{i}{2}{\rm Tr}\,(G_\Sigma\mathcal{M}')^2 - i\sum_{n=3}^{\infty}\frac{1}{n}{\rm Tr}\,(-G_\Sigma\mathcal{M}')^n\Big\}. \end{aligned} \tag{24}$$

The vanishing of the second variation of this effective action with respect to $\mathcal{M}'$,

$$\left.\frac{\delta^2 W_{\rm eff}}{\delta\mathcal{M}'\delta\mathcal{M}'}\right|_{\mathcal{M}'=0}\cdot\Gamma = 0\ ,$$

leads to the homogeneous Bethe–Salpeter equation for the vertex function, Γ,

$$\Gamma = -iK(G_\Sigma\Gamma G_\Sigma). \tag{25}$$

This equation describes the bound state spectrum. It corresponds to the usual Bethe–Salpeter equation in ladder approximation.

Given the solutions of eqs.(22, 25) describing the bound state spectrum, one can calculate matrix elements between on-shell bound states describing meson decays, semi-leptonic processes, *etc.*, in the framework of the effective meson action, eq.(20). The bosonized action summarizes these processes in a concise way and greatly facilitates the evaluation of the corresponding matrix elements. One formally expands the bilocal field, $\mathcal{M}$, in bound state amplitudes,

$$\begin{aligned} \mathcal{M}(x,y) \;&=\; \mathcal{M}\Big(x-y\,\Big|\,\frac{x+y}{2}\Big) \;=\; \sum_H\int\frac{d\mathcal{P}^3}{(2\pi)^{3/2}\sqrt{2\omega_H}} \\ &\times\int\frac{d^4q}{(2\pi)^4}e^{iq(x-y)}\Big\{e^{i\mathcal{P}\frac{x+y}{2}}a_H^+(\vec{\mathcal{P}})\Gamma_H(q|\mathcal{P}) + e^{-i\mathcal{P}\frac{x+y}{2}}a_H(\vec{\mathcal{P}})\overline{\Gamma}_H(q|\mathcal{P})\Big\}. \end{aligned} \tag{26}$$

The sum runs over the set of quantum numbers, H, of hadrons contributing to the bilocal field. Here, the individual bound states have mass M_H and total 4–momentum $\mathcal{P} = (\omega_H, \vec{\mathcal{P}})$, with $\omega_H(\vec{\mathcal{P}}) = (\vec{\mathcal{P}}^2 + M_H^2)^{1/2}$. The amplitudes $\Gamma_H(q|\mathcal{P})$ and $\overline{\Gamma}_H(q|\mathcal{P}) = \Gamma_H(q|-\mathcal{P})$ are on–shell solutions of the Bethe–Salpeter equation, eq.(25), with 3–momentum $\vec{\mathcal{P}}$. The coefficients $a_H^+(\vec{\mathcal{P}}), a_H(\vec{\mathcal{P}})$ may then be interpreted as creation

and annihilation operators of physical (on–shell) mesons, and matrix elements can be calculated as usual. In particular, the bound states amplitudes are normalized by the requirement that the matrix element of the quadratic (free) part of the effective action, eq.(20),

$$W_{\text{eff}}^{(2)} \;=\; -i\tfrac{1}{2}N_c \mathrm{Tr}\,(G_\Sigma \mathcal{M})^2 \tag{27}$$

have the normalization corresponding to a physical (elementary) particle. This is simply the statement of the correct relativistic dispersion law of the c.o.m. motion of the bound state[2].

In order to describe meson weak decays, we couple the theory eq.(1) to an external weak leptonic current. The corresponding lagrangian at quark level is given by

$$\mathcal{L}_{\text{semi}} \;=\; \frac{G_F}{\sqrt{2}}\{V_{ij}(\bar{Q}_i(x)O_\mu q_j(x))l_\mu(x) \;+\; \text{h.c.}\}. \tag{28}$$

Here, l_μ is the leptonic current,

$$\begin{aligned} l_\mu(x) \;&\equiv\; \bar{l}(x)O_\mu \nu_l(x), \qquad l \;=\; (e,\mu,\tau), \qquad \nu_l \;=\; (\nu_e,\nu_\mu,\nu_\tau), \\ O_\mu \;&=\; \gamma_\mu(1+\gamma_5), \end{aligned} \tag{29}$$

V_{ij} are the elements of the Kobayashi–Maskawa matrix, and G_F is the Fermi constant. In eq.(28), Q denotes the column of (u,c,t)–quarks, q the (d,s,b)–quarks. On the level of the effective meson action, eq.(20), the electroweak coupling can be incorporated by shifting the bilocal field by the local leptonic current,

$$\mathcal{M}_{ij}(x,y) \;\to\; \mathcal{M}_{ij}(x,y) \;+\; \hat{L}_{ij}(x,y), \tag{30}$$

$$\hat{L}_{ij}(x,y) \;=\; \frac{G_F}{\sqrt{2}}\delta^4(x-y)V_{ij}O^\mu l_\mu(x)e^{i\mathcal{P}_L\frac{x+y}{2}}, \tag{31}$$

where $\mathcal{P}_L$ is the momentum of the leptonic pair. After this shift, the part of eq.(27) describing meson decay into leptons is

$$W_{\text{semi}}^{(2)} \;=\; -iN_c \mathrm{Tr}\,(G_\Sigma \mathcal{M} G_\Sigma \hat{L}). \tag{32}$$

Semileptonic decays of mesons, which for heavy–light mesons are parametrized in terms of the Isgur–Wise functions [32], are mediated by

$$W_{\text{semi}}^{(3)} \;=\; iN_c \mathrm{Tr}\,(G_\Sigma \mathcal{M} G_\Sigma \mathcal{M} G_\Sigma \hat{L}). \tag{33}$$

In the following we consider in detail the Schwinger–Dyson–equation, eqs.(22), and the Bethe–Salpeter–equation, eq.(25), for a covariant instantaneous interaction, eq.(10). In particular, bound states will be described by 3–dimensional wave functions. The evaluation of meson observables then proceeds in general as follows. Given

[2]For a covariant instantaneous interaction, eq.(10), the interaction does not contribute to the normalization of the bound state, *cf.* appendix B.

the solution for the bound state wave function in the rest frame, one can reconstruct the Bethe–Salpeter amplitude in the rest frame, *cf.* eqs.(56, 58, 60) below. By virtue of the relativistic formulation of the interaction, eq.(10), the bound state amplitude transforms covariantly and may be boosted to an arbitrary velocity. Matrix elements of the vertices eqs.(32, 33) *etc.* between moving bound states can then be calculated from the expansion of the bilocal field, eq.(26). The evaluation of the matrix element for pseudoscalar meson decay in this approach is discussed in appendix B.

3.2 The quark spectrum within a meson

For an instantaneous interaction of the form eq.(10) the Schwinger–Dyson equation, eq.(22), describes the spectrum of quark quasiparticles in the rest frame of the meson. In momentum space it takes the form

$$\Sigma(p^{\perp}) \;=\; \hat{m}^0 - i\int \frac{d^4q}{(2\pi)^4} V(p^{\perp}-q^{\perp})\not\eta G_{\Sigma}(q)\not\eta. \tag{34}$$

Here, the 4–dimensional Fourier transform of the interaction kernel, eq.(10), depends only on the transverse part of the relative momenta,

$$V(p^{\perp}-q^{\perp}) \;=\; \int d^4x\, e^{-i(p-q)x} V(x^{\perp})\delta(x\cdot\eta). \tag{35}$$

In the following we consider eq.(34) in the rest frame, where $\eta_\mu = (1,\vec{0})$, $\not\eta = \gamma^0$, and eq.(35) coincides with the usual 3–dimensional Fourier transform. Assuming that $\Sigma(\mathbf{p})$ is diagonal in flavor, $\Sigma = \mathrm{diag}\,(\Sigma_1, \ldots \Sigma_{N_f})$, eq.(34) splits into identical equations for the Σ_n with bare mass m_n^0, $n = 1, \ldots N_f$. We shall omit the flavor index on Σ in the following. The quark self–energy in the rest frame has a scalar and a vector part,

$$\Sigma(\mathbf{p}) \;=\; A(|\mathbf{p}|)\,|\mathbf{p}| \,+\, B(|\mathbf{p}|)\,\mathbf{p}\cdot\vec{\gamma}. \tag{36}$$

In the following it will be convenient to make a polar decomposition of the quark energy in the form

$$\Sigma(\mathbf{p}) \,+\, \mathbf{p}\cdot\vec{\gamma} \;=\; E(|\mathbf{p}|)\,S^2(\mathbf{p}), \tag{37}$$

where $S^2(\mathbf{p})$ is the square of a Foldy–Wouthuysen matrix,

$$S^{\pm 2}(\mathbf{p}) \;=\; \sin\phi(|\mathbf{p}|) \,\pm\, \hat{\mathbf{p}}\cdot\vec{\gamma}\cos\phi(|\mathbf{p}|), \tag{38}$$

$$E(|\mathbf{p}|)\sin\phi(|\mathbf{p}|) \;=\; A(|\mathbf{p}|)\,|\mathbf{p}|, \qquad E(|\mathbf{p}|)\cos\phi(|\mathbf{p}|) \;=\; (1+B(|\mathbf{p}|))\,|\mathbf{p}|. \tag{39}$$

Here, $0 \le \phi(|\mathbf{p}|) \le \frac{1}{2}\pi$ is the so–called chiral angle. (In the quasiparticle language this angle defines the rotation of the massive quasiparticle spinors relative to the free quark spinors in the vacuum of broken chiral symmetry [14, 15].) In this parametrization the

quark propagator, $G_\Sigma(q)$, becomes

$$G_\Sigma(q) \;=\; \frac{1}{\not{q} - \Sigma(\mathbf{q})} \;=\; \Big(\frac{\Lambda_+(\mathbf{q})}{q_0 - E(\mathbf{q}) + i\epsilon} + \frac{\Lambda_-(\mathbf{q})}{q_0 + E(\mathbf{q}) - i\epsilon}\Big)\gamma_0$$
$$\;=\; \gamma_0\Big(\frac{\overline{\Lambda}_+(\mathbf{q})}{q_0 - E(\mathbf{q}) + i\epsilon} + \frac{\overline{\Lambda}_-(\mathbf{q})}{q_0 + E(\mathbf{q}) - i\epsilon}\Big). \tag{40}$$

Here, the matrices $\Lambda_\pm(\mathbf{q})$, $\overline{\Lambda}_\pm(\mathbf{q})$ are defined as

$$\Lambda_\pm(\mathbf{q}) \;=\; S^{-1}(\mathbf{q})\,\overset{0}{\Lambda}_\pm\,S(\mathbf{q}), \qquad \overline{\Lambda}_\pm(\mathbf{q}) \;=\; S(\mathbf{q})\,\overset{0}{\Lambda}_\pm\,S^{-1}(\mathbf{q}), \tag{41}$$

where

$$\overset{0}{\Lambda}_\pm \;=\; \tfrac{1}{2}(1 \pm \gamma_0) \tag{42}$$

is the usual projector on positive and negative energy components. The first power of the Foldy–Wouthuysen matrices, $S^{\pm 1}(\mathbf{q})$, can be expressed from eq.(38) as

$$S^{\pm 1}(\mathbf{q}) \;=\; \cos\nu(|\mathbf{q}|) \,\pm\, \hat{\mathbf{q}}\cdot\vec{\gamma}\sin\nu(|\mathbf{q}|), \qquad \nu(|\mathbf{q}|) \;=\; \tfrac{1}{2}(\tfrac{1}{2}\pi - \phi(|\mathbf{q}|)). \tag{43}$$

Inserting the quark propagator in the form of eq.(40) into the Schwinger–Dyson equation, eq.(34), and taking traces one obtains a system of equations for $E(|\mathbf{p}|)$ and $\phi(|\mathbf{p}|)$,

$$\begin{aligned} E(|\mathbf{p}|)\sin\phi(|\mathbf{p}|) &= m^0 - \tfrac{1}{2}\int\frac{d^3q}{(2\pi)^3}V(\mathbf{p}-\mathbf{q})\sin\phi(|\mathbf{q}|), \\ E(|\mathbf{p}|)\cos\phi(|\mathbf{p}|) &= |\mathbf{p}| - \tfrac{1}{2}\int\frac{d^3q}{(2\pi)^3}V(\mathbf{p}-\mathbf{q})\hat{\mathbf{p}}\cdot\hat{\mathbf{q}}\cos\phi(|\mathbf{q}|). \end{aligned} \tag{44}$$

These equations define the single-particle spectrum of two quarks forming a bound state. From eqs.(44) one obtains an integral equation for the chiral angle,

$$m^0\cos\phi(|\mathbf{p}|) - |\mathbf{p}|\sin\phi(|\mathbf{p}|) \;=$$
$$\tfrac{1}{2}\int\frac{d^3q}{(2\pi)^3}V(\mathbf{p}-\mathbf{q})\,\left[\cos\phi(|\mathbf{p}|)\sin\phi(|\mathbf{q}|) - \hat{\mathbf{p}}\cdot\hat{\mathbf{q}}\,\sin\phi(|\mathbf{p}|)\cos\phi(|\mathbf{q}|)\right]. \tag{45}$$

Here, $\phi(|\mathbf{p}|)$ satisfies the boundary conditions $\phi(0) = \frac{1}{2}\pi$ and $\phi(|\mathbf{p}|) \to 0$ for $|\mathbf{p}| \to \infty$. From the chiral angle, the quark energy can be obtained as

$$E(|\mathbf{p}|) \;=\; m^0\sin\phi(|\mathbf{p}|) + |\mathbf{p}|\cos\phi(|\mathbf{p}|)$$
$$-\tfrac{1}{2}\int\frac{d^3q}{(2\pi)^3}V(\mathbf{p}-\mathbf{q})\,\left[\sin\phi(|\mathbf{p}|)\sin\phi(|\mathbf{q}|) + \hat{\mathbf{p}}\cdot\hat{\mathbf{q}}\,\cos\phi(|\mathbf{p}|)\cos\phi(|\mathbf{q}|)\right]. \tag{46}$$

We remark that absolute values of the quark energy may be shifted by a constant amount, E_0, without affecting the bound state spectrum, by adding to the potential a contact term, $V_0(\mathbf{p}-\mathbf{q}) = -E_0(2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{q})$, *cf.* the discussion in [16, 15].

If the potential contains a Coulomb interaction, $V_{\rm C}(\mathbf{p}-\mathbf{q}) = -\frac{4}{3}\alpha_s/(\mathbf{p}-\mathbf{q})^2$, the integrals in eqs.(44) are ultraviolet divergent and require renormalization. Consequently, we perform in eqs.(44) a wave function and mass renormalization and replace the bare quark kinetic energy and mass, $|\mathbf{p}|$ and m^0, by

$$|\mathbf{p}| \;\; \rightarrow \;\; |\mathbf{p}| \; + \; \tfrac{1}{2} \int \frac{d^3q}{(2\pi)^3} V_{\rm C}(\mathbf{p}-\mathbf{q})\, \hat{\mathbf{p}}\cdot\hat{\mathbf{q}}, \tag{47}$$

$$m^0 \;\; \rightarrow \;\; m^0 \; + \; \tfrac{1}{2} \int \frac{d^3q}{(2\pi)^3} V_{\rm C}(\mathbf{p}-\mathbf{q}) \frac{m^0}{\sqrt{|\mathbf{q}|^2 + m^{0\,2}}}. \tag{48}$$

The equations resulting from eqs.(44, 45, 46) upon this substitution are finite. The boundary conditions on the chiral angle remain unchanged. Furthermore, with the definitions eqs.(47, 48) the single–particle energy for $|\mathbf{p}| \rightarrow \infty$ behaves like that of a free quark. In the case of an asymptotically free potential, $\alpha_s = \alpha_s(|\mathbf{p}|)$ it has been shown that the Schwinger–Dyson equation can be renormalized consistently by requiring that the quark propagator reduce to the free propagator at some large 3–momentum $\mathbf{p}_{\rm ren}$, with $|\mathbf{p}_{\rm ren}| \rightarrow \infty$ [16, 17]. Here, we are working with a potential, the UV–divergent part of which is of pure Coulomb form, α_s = const., so that the definition eq.(47) is sufficient[3]. The linear potential in eq.(16) is UV–finite and does not contribute to the renormalization. The redefinition of m^0, eq.(48), is not a renormalization in the strict sense, as it corresponds to using a momentum–dependent renormalization constant. In the framework of this potential model we take the point of view that eq.(48) is simply a prescription to softly break chiral symmetry for the light flavors with parameter m^0. Hence the value of m^0 is not inferred from the QCD current mass but may be determined phenomenologically *e.g.* by fitting the pion mass. This is usual practice in the framework of the Nambu–Jona-Lasinio model [2, 3]. Definition eq.(48) leads to a smooth behavior of the pion mass and decay constant in the chiral limit, as can be seen from figs. 4 and 5. For heavy quarks, we will use the approximation of a constant constituent quark mass, *cf.* below.

The ratio of the scalar to vector part of the quark self–energy,

$$m(|\mathbf{p}|) \;\; = \;\; \frac{A(|\mathbf{p}|)\,|\mathbf{p}|}{1 + B(|\mathbf{p}|)} \;\; = \;\; |\mathbf{p}| \tan\phi(|\mathbf{p}|), \tag{49}$$

can be interpreted as a momentum–dependent "constituent" quark mass, which reduces to the current quark mass, m^0, in the limit $|\mathbf{p}| \rightarrow \infty$ [17].

Eq.(45) constitutes a non-linear integral equation for the chiral angle, $\phi(|\mathbf{p}|)$. By performing the angular integral one obtains a 1–dimensional equation, which we solve numerically using the relaxation methods of Adler *et al.* [16, 33].

Numerical solutions of the Schwinger–Dyson equation for the chiral angle, $\phi(|\mathbf{p}|)$, and the quark energy, $E(|\mathbf{p}|)$, for a light quark flavor are shown in figs. 1 and 2. Here,

[3]For a pure Coulomb potential without a mass scale the definition eq.(47) falls within the scope of multiplicative renormalization. In other words, for a pure Coulomb potential the subtraction procedure of Finger and Mandula [14] coincides with the renormalization of Adler and Davis [16].

the potential is the one used below to describe D– and B–mesons ($\sigma = 0.41\,\mathrm{GeV}, \alpha_s = 0.39$). Note that the attractive interaction causes the quark energy to become negative at small momenta. This fact is intimately related to the spontaneous breaking of chiral symmetry and the emergence of Goldstone bosons [15]. The dynamical quark mass, $m(|\mathbf{p}|)$, is shown in fig.3. We also show the corresponding quantities for the oscillator model of [15] ($V_0 = 0.247\,\mathrm{GeV}$), for which eq.(45) reduces to a differential equation for $\phi(|\mathbf{p}|)$.

For heavy quark flavors, the bare quark mass, m^0, is much larger than the dynamically generated mass. One may then neglect the dynamical contribution to the mass and approximate $\Sigma(\mathbf{q})$ by a constant constituent quark mass, $\Sigma(\mathbf{p}) \equiv m_Q$. In this case we have in eq.(39) $A(|\mathbf{p}|)|\mathbf{p}| = m_Q,\ B(|\mathbf{p}|) = 0$, so that

$$\phi(|\mathbf{p}|) \;=\; \arctan\frac{m_Q}{|\mathbf{p}|}, \qquad E(|\mathbf{p}|) \;=\; \sqrt{|\mathbf{p}|^2 + m_Q^2}, \qquad \nu(|\mathbf{p}|) \;=\; \tfrac{1}{2}\arctan\frac{|\mathbf{p}|}{m_Q}. \quad (50)$$

The chiral angle and the quark energy for a heavy quark are also shown in figs. 1 and 2. A constant quark mass is also frequently used to describe the light–quark sector, see *e.g.* [34]. We will comment on the consequences of this approximation below.

In the heavy–quark limit, if m_Q becomes much larger than the momentum of the relative motion of the heavy–light bound state, one may approximate the Foldy–Wouthuysen angle of the heavy flavor by its value at $|\mathbf{p}| = 0$, $\phi(|\mathbf{p}|) \equiv \frac{1}{2}\pi$, $\nu(|\mathbf{p}|) \equiv 0$. Furthermore, the heavy–quark energy becomes $E(\mathbf{p}) \sim m_Q + \frac{1}{2}|\mathbf{p}|^2/m_Q \sim m_Q + O(1/m_Q)$; to leading order one can neglect the heavy–quark kinetic energy. In other words, in this approach the heavy–quark limit amounts to neglecting the variations of the heavy–quark energy and chiral angle over the range of momenta contributing to the bound–state wave function.

3.3 Equations for bound state wave functions

Given the quark single–particle spectrum in the vacuum of the effective theory, we now turn to the description of meson bound states. For the covariantly written potential kernel, eq.(10), the Bethe–Salpeter amplitude, $\Gamma(p|\mathcal{P})$, depends only on the transverse part of the internal momentum, $p^\perp$. With this interaction the Bethe–Salpeter equation, eq.(25), for a bound state of flavor $q_i\bar{q}_j$ in an arbitrary frame reads

$$\Gamma(p^\perp|\mathcal{P}) \;=\; i\int\frac{d^4q}{(2\pi)^4}V(p^\perp - q^\perp)\,\not{\eta}G_i(q + \tfrac{1}{2}\mathcal{P})\Gamma(q^\perp|\mathcal{P})G_j(q - \tfrac{1}{2}\mathcal{P})\not{\eta}. \quad (51)$$

Here, $G_n \equiv G_{\Sigma_n}$ is the quark propagator of flavor n defined in eq.(40), and $\mathcal{P}$ denotes the total momentum of the bound state. For an instantaneous interaction it is convenient to describe the bound state in terms of a Salpeter wave function,

$$\Psi(q^\perp|\mathcal{P}) \;=\; i\int\frac{dq^{||}}{2\pi}G_i(q + \tfrac{1}{2}\mathcal{P})\Gamma(\mathbf{q}|\mathcal{P})G_j(q - \tfrac{1}{2}\mathcal{P}). \quad (52)$$

where $q^{||} = q\cdot\eta$. In the following we shall restrict ourselves to the rest frame, where $\mathcal{P} = (M, \vec{0})$, with M the bound state mass, and $q^{||} = q_0$. We drop the argument $\mathcal{P}$ in $\Gamma(\mathbf{p}|\mathcal{P}), \Psi(\mathbf{p}|\mathcal{P})$ in the following. Performing in eq.(51) the integral over q_0 one obtains for $\Psi(\mathbf{p})$ the equation

$$\Psi(\mathbf{p}) \;=\; \frac{\Pi_{+-}(\mathbf{p})}{E_p - M} + \frac{\Pi_{-+}(\mathbf{p})}{E_p + M}. \tag{53}$$

Here, $E_p = E_i(|\mathbf{p}|) + E_j(|\mathbf{p}|)$ is the sum of the single–particle energies for the two quark flavors, and

$$\Pi_{\pm\mp}(\mathbf{p}) \;=\; \Lambda_{i\pm}(\mathbf{p})\gamma_0\Gamma(\mathbf{p})\gamma_0\overline{\Lambda}_{j\mp}(\mathbf{p}), \tag{54}$$

$$\Gamma(\mathbf{p}) \;=\; \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}-\mathbf{q})\gamma_0\Psi(\mathbf{q})\gamma_0, \tag{55}$$

with the projectors $\Lambda_{n\pm}(\mathbf{p}), \overline{\Lambda}_{n\mp}(\mathbf{p})$ for flavor n defined in eq.(41). Rather than working with eq.(53) directly it is more suitable to introduce an "undressed" wave function,

$$\overset{0}{\Psi}(\mathbf{p}) \;=\; S_i(\mathbf{p})\Psi(\mathbf{p})S_j(\mathbf{p}). \tag{56}$$

Here, $S_n(\mathbf{p})$ are the Foldy–Wouthuysen matrices, eq.(43), corresponding to the two quark flavors. The new wave function satisfies an equation analogous to eq.(53),

$$\overset{0}{\Psi}(\mathbf{p}) \;=\; \frac{\overset{0}{\Pi}_{+-}(\mathbf{p})}{E_p - M} + \frac{\overset{0}{\Pi}_{-+}(\mathbf{p})}{E_p + M}, \tag{57}$$

where $\overset{0}{\Pi}_{\pm\mp}(\mathbf{p})$ now involves the free projectors, eq.(42),

$$\overset{0}{\Pi}_{\pm\mp}(\mathbf{p}) \;=\; -\,\overset{0}{\Lambda}_{\pm}\, S_i^{-1}(\mathbf{p})\Gamma(\mathbf{p})S_j^{-1}(\mathbf{p})\,\overset{0}{\Lambda}_{\mp}\,. \tag{58}$$

As a consequence, the new wave function satisfies the constraints

$$\overset{0}{\Lambda}_{\pm}\overset{0}{\Psi}(\mathbf{p})\,\overset{0}{\Lambda}_{\pm} \;=\; 0. \tag{59}$$

By applying projection operators, eq.(57) can be written in the form of a Schrödinger equation,

$$[E_p \mp M]\,\overset{0}{\Lambda}_{\pm}\overset{0}{\Psi}(\mathbf{p})\,\overset{0}{\Lambda}_{\mp} \;=\; \overset{0}{\Pi}_{\pm\mp}(\mathbf{p})\,. \tag{60}$$

It has the structure of a generalized eigenvalue equation for the bound state wave function, $\overset{0}{\Psi}(\mathbf{p})$, with M as eigenvalue[4]. We now decompose $\overset{0}{\Psi}(\mathbf{p})$ as

$$\overset{0}{\Psi}(\mathbf{p}) \;=\; \overset{0}{\Psi}{}^{(1)}(\mathbf{p}) + \gamma_0\cdot\overset{0}{\Psi}{}^{(2)}(\mathbf{p}). \tag{61}$$

[4]The mathematical structure of the instantaneous Bethe–Salpeter equation has recently been analyzed in [34, 35].

Because of the constraint, eq.(59), we can expand the functions $\overset{0}{\Psi}{}^{(k)}(\mathbf{p})$ completely in the set of Dirac matrices $\{\gamma_5,\, i\vec{\gamma}\}$. We write

$$\overset{0}{\Psi}{}^{(k)}(\mathbf{p}) \;=\; L^{(k)}(\mathbf{p})\,\gamma_5 \,+\, i\mathbf{N}^{(k)}(\mathbf{p})\cdot\vec{\gamma} \qquad (k=1,2). \tag{62}$$

By taking traces of eq.(60) one obtains after a straightforward but lengthy calculation a system of equations for the component functions, $L^{(k)}(\mathbf{p}), \mathbf{N}^{(k)}(\mathbf{p})$, $k = 1,2$. In calculating the traces it is convenient to introduce a dreibein in momentum space, $\{\hat{\mathbf{p}},\, \hat{e}^1_{\mathbf{p}},\, \hat{e}^2_{\mathbf{p}}\}$, with $\hat{\mathbf{p}}\cdot\hat{e}^a_{\mathbf{p}} = 0$, $\hat{e}^a_{\mathbf{p}}\cdot\hat{e}^b_{\mathbf{p}} = \delta^{ab}$ $(a,b=1,2)$ and expand the wave function in the set $\{\gamma_5,\, i\hat{e}^a_{\mathbf{p}}\cdot\vec{\gamma},\, i\hat{\mathbf{p}}\cdot\vec{\gamma}\}$. We find

$$ML^{\binom{2}{1}}(\mathbf{p}) \;-\; E_p L^{\binom{1}{2}}(\mathbf{p}) \;=\; \int\frac{d^3q}{(2\pi)^3}V(\mathbf{p}-\mathbf{q}) \tag{63}$$
$$\times\Big\{\big[c^{\mp}_p c^{\mp}_q \,+\, s^{\mp}_p s^{\mp}_q\,\hat{\mathbf{p}}\cdot\hat{\mathbf{q}}\big]\,L^{\binom{1}{2}}(\mathbf{q}) \;+\; s^{\mp}_p s^{\pm}_q\,\hat{\mathbf{p}}\cdot\big(\hat{\mathbf{q}}\times\mathbf{N}^{\binom{2}{1}}(\mathbf{q})\big)\Big\}$$

$$M\mathbf{N}^{\binom{2}{1}}(\mathbf{p}) \;-\; E_p\mathbf{N}^{\binom{1}{2}}(\mathbf{p}) \;=\; \int\frac{d^3q}{(2\pi)^3}V(\mathbf{p}-\mathbf{q}) \tag{64}$$
$$\times\Big\{\big[c^{\mp}_p c^{\mp}_q\,P^T_{\mathbf{p}}P^T_{\mathbf{q}} + c^{\mp}_p c^{\pm}_q\,P^T_{\mathbf{p}}P^L_{\mathbf{q}} + c^{\pm}_p c^{\mp}_q\,P^L_{\mathbf{p}}P^T_{\mathbf{q}} + c^{\pm}_p c^{\pm}_q\,P^L_{\mathbf{p}}P^L_{\mathbf{q}}$$
$$-s^{\mp}_p s^{\mp}_q\,\hat{\mathbf{p}}\times(\hat{\mathbf{q}}\times\;.\;) + s^{\pm}_p s^{\pm}_q\,\hat{\mathbf{p}}\,(\hat{\mathbf{q}}\cdot\;.\;)\big]\,\mathbf{N}^{\binom{1}{2}}(\mathbf{q})$$
$$-s^{\pm}_p s^{\mp}_q\,\hat{\mathbf{p}}\times\hat{\mathbf{q}}\,L^{\binom{2}{1}}(\mathbf{q})\Big\}$$

Here, $P^T_{\mathbf{p}}, P^L_{\mathbf{p}}$ are the transverse and longitudinal 3–dimensional projectors,

$$P^L_{\mathbf{p}} \;=\; \hat{\mathbf{p}}\otimes\hat{\mathbf{p}}, \qquad P^T_{\mathbf{p}} \;=\; 1 - P^L_{\mathbf{p}} \;=\; -\hat{\mathbf{p}}\times(\hat{\mathbf{p}}\times\,.\,), \tag{65}$$

with a similar definition for $\hat{\mathbf{q}}$. The factors $c^{\pm}_p, s^{\pm}_p$ are the result of the Foldy–Wouthuysen transformation, eq.(56). We have introduced the short–hand notation

$$\begin{aligned} c^{\pm}_p &= \cos\nu_{ip}\cos\nu_{jp} \mp \sin\nu_{ip}\sin\nu_{jp} \;=\; \cos(\nu_{ip}\pm\nu_{jp}), \\ s^{\pm}_p &= \sin\nu_{ip}\cos\nu_{jp} \pm \cos\nu_{ip}\sin\nu_{jp} \;=\; \sin(\nu_{ip}\pm\nu_{jp}), \end{aligned} \tag{66}$$

with a similar definition for $c^{\pm}_q, s^{\pm}_q$. The angle $\nu_{np} \equiv \nu_n(|\mathbf{p}|)$ for flavor $n = i,j$ has been defined in eq.(43). The functions $E_p = E_i(|\mathbf{p}|) + E_j(|\mathbf{p}|)$ and ν_{ip}, ν_{jp} contain the entire information on the single–particle spectrum. They have to be provided either as solutions of the Schwinger–Dyson equation for the quark self energy, eq.(44), or by the approximation of a constant quark mass, eq.(50).

The equations eqs.(63, 64) for the bound state wave functions have a very compact form. Note in particular that the pseudoscalar–axial and scalar–vector meson mixing is taken into account here. Corresponding equations have been given by Le Yaouanc

et al. for the case of equal quark masses and a harmonic oscillator potential [21]. Through the use of the Foldy–Wouthuysen transformation, eq.(56), we have preserved the simple structure of the wave function, eq.(59), even in the general case of unequal quark flavors. Note that in the case of identical quark flavors (isospin limit), one has $s_p^- = 0$ in eq.(66), and the equations for the L– and $\mathbf{N}$–component decouple[5]. We remark that equations in a parametrization somewhat different from to that of eqs.(63, 64) have been derived by Lagaë using a variational approach [35]. The use of the Foldy–Wouthuysen transformation in the form eq.(56) provides a simple and more transparent alternative.

The bound state wave functions are normalized by condition eq.(B.8), which fixes the relativistic dispersion law for the bound state. From eqs.(56, 58, 60) the Bethe–Salpeter amplitude can be reconstructed from the "undressed" wave function in the rest frame. The wave functions contain the full information about the bound state on its mass shell. In appendix B, the pseudoscalar meson decay matrix element is evaluated in terms of the wave function.

For calculation of the meson spectrum it is necessary to decompose eqs.(63, 64) into equations for bound states of given angular momentum and parity in the rest frame. This is done in appendix A.1. The resulting radial equations are solved numerically using the Multhopp method [24]. In the presence of the momentum–dependent Foldy–Wouthuysen factors in the integral equation this momentum–space method is much more convenient than the commonly used matrix methods, in which the matrix elements of the potential are evaluated in position space [34, 35]. In appendix A.2 we show how the Multhopp method can be adapted so that essentially no complications arise from the Foldy–Wouthuysen factors.

4 Light–light and heavy–light mesons

The bound state equations derived from the bilocal effective action, eq.(20), describe mesons for arbitrary quark masses. We now apply this description to the study of light–light and heavy–light mesons. Before embarking on the numerical solution of the Salpeter equations it is worthwhile to investigate some limiting cases. In particular, we wish to demonstrate how eqs.(63, 64) describe both pseudoscalar Goldstone bosons in the chiral limit, $m^0 \to 0$, and degenerate heavy–light mesons in the heavy–quark limit. Let us first consider eq.(63) in the isospin limit with two light quark flavors, *i.e.*, for the pion. In this case one has $\nu_{ip} = \nu_{jp} \equiv \nu(|\mathbf{p}|) = \frac{1}{2}(\frac{1}{2}\pi - \phi(|\mathbf{p}|))$, so that

$$\begin{array}{rclcrcl} c_p^+ &=& \sin\phi(|\mathbf{p}|) & \qquad & s_p^+ &=& \cos\phi(|\mathbf{p}|) \\ c_p^- &=& 1 & & s_p^- &=& 0 \end{array} \tag{67}$$

with $\phi(|\mathbf{p}|)$ the solution of eq.(44). If furthermore $m^0 = 0$, comparison of the Salpeter equation, eq.(63), with the Schwinger–Dyson equation, eq.(44), shows that eq.(63)

[5] The equations for wave functions written down in [13] apply only to this case of equal quark flavors, since the L–$\mathbf{N}$ mixing term has been omitted there.

possesses a massless Goldstone mode solution of the form

$$L^{(1)}(\mathbf{p}) \;\; \propto \;\; \sin\phi(|\mathbf{p}|), \qquad L^{(2)}(\mathbf{p}) \; = \; 0, \qquad M \; = \; 0. \tag{68}$$

For this property it is crucial that chiral symmetry is broken spontaneously by the same interactions which bind the quarks in the meson, *i.e.*, that the quark self–energy is taken as the solution of the Schwinger–Dyson equation and not approximated by a constant constituent quark mass [31]. Note also that the pseudoscalar wave function is normalizable only for $m^0 > 0$.

Of particular interest is the case of a small but finite current quark mass, the chiral limit. Specifically, we want to see how the pseudoscalar meson mass and decay constant behaves in the limit $m^0 \to 0$. We thus solve the Schwinger–Dyson equation, eq.(44), and the pseudoscalar bound state equation, eqs.(63, A.7), for small current quark masses, $m_u^0 = m_d^0 = m^0$. Results for the linear plus Coulomb potential as well as the oscillator model are shown in figs. 4 and 5. As can be seen from fig.4, the pion mass vanishes in the chiral limit like $m_\pi^2 \propto m^0$, in accordance with current algebra. The pion decay constant, f_π, is shown in fig.5. As expected, it approaches a finite limit if $m^0 \to 0$. We remark that for a pure oscillator potential ($V_0 = 0.247\,\mathrm{GeV}$) we reproduce in the chiral limit the value of Le Yaouanc *et al.*, $f_\pi = 20\,\mathrm{MeV}$, while for a pure linear potential ($\sigma = 0.40\,\mathrm{GeV}$) we find $f_\pi = 0.11\,\mathrm{MeV}$, which is in agreement with the value obtained by Adler and Davis [16]. Pion properties in the chiral limit have also been studied by Alkofer and Amundsen using the inhomogeneous Bethe–Salpeter equation [17]. For an instantaneous interaction, the wave function description of bound states, eq.(52), is a convenient alternative, especially for the study of excited states. Also shown in fig.5 is the decay constant for the radially excited pion state, which vanishes in the chiral limit, as expected on general grounds [21]. The reason for this is that as the pion wave function approaches $\sin\phi(|\mathbf{p}|)$, the excited state wave functions become orthogonal to $\sin\phi(|\mathbf{p}|)$ and thus the integral for the pion decay constant, eq.(B.15), vanishes. The pion radial wave function, *cf.* eq.(A.3), is shown in fig.6, for a current mass of $m^0 = 1\,\mathrm{MeV}$.

It is well–known that a charmonium (linear plus Coulomb) potential with usual parameters underestimates the strength of spontaneous chiral symmetry breaking, which manifests itself in the too small value of f_π [16]. Improved values can be obtained by including either an intermediate–range attractive potential [30] or transverse gluon exchange [17].

Thus, we have verified that the bilocal effective meson action, eq.(20), reproduces the successful phenomenology of the pion as a Goldstone boson, if the breaking of chiral symmetry is described consistently with the interactions which form the bound state.

Let us now consider heavy–light mesons. Specifically, we shall investigate a bound state of a heavy quark and a light antiquark ($Q_i\bar{q}_j$). For the heavy quark we neglect spontaneous chiral symmetry breaking and approximate its energy by a constant mass, *cf.* eq.(50). In order to take the limit $m_Q \to \infty$, we subtract from E_p and M in eqs.(63, 64) the heavy quark mass, *i.e.*, we consider binding energies relative to the heavy quark mass. For the light flavor, we take take into account the dynamical breaking of chiral

symmetry and take $E_j(|\mathbf{p}|)$, $\nu_j(|\mathbf{p}|)$ as the solution of the Schwinger–Dyson equation, eq.(44).

In particular, in the heavy–quark limit, if m_Q becomes much larger than the range of momenta contributing to the integrals over the bound–state wave function we may simply take $\nu_i(|\mathbf{p}|) \equiv 0$. In this case we have

$$\begin{array}{rclclcl} c_p^{\pm} & = & \cos(\pm\nu_{jp}) & = & \cos(\nu_{jp}) & \equiv & c_p, \\ s_p^{\pm} & = & \sin(\pm\nu_{jp}) & = & \pm\sin(\nu_{jp}) & \equiv & \pm s_p, \end{array} \tag{69}$$

while $E_i(|\mathbf{p}|) = \sqrt{m_Q^2 + |\mathbf{p}|^2} = m_Q + O(1/m_Q)$. Consequently, in eqs.(63, 64) the equations for the (1)– and (2)–components of the wave function coincide, and the solutions satisfy

$$L^{(1)}(\mathbf{p}) \;=\; L^{(2)}(\mathbf{p}) \;\equiv\; L(\mathbf{p}), \qquad \mathbf{N}^{(1)}(\mathbf{p}) \;=\; \mathbf{N}^{(2)}(\mathbf{p}) \;\equiv\; \mathbf{N}(\mathbf{p}). \tag{70}$$

Eqs.(63, 64) thus simplify to

$$\begin{aligned} (M - E_p)L(\mathbf{p}) \;&=\; \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}-\mathbf{q}) \qquad\qquad (71) \\ &\times \Big\{ [c_p c_q + s_p s_q\, \hat{\mathbf{p}}\!\cdot\!\hat{\mathbf{q}}]\, L(\mathbf{q}) \;-\; s_p s_q\, \hat{\mathbf{p}}\!\cdot\!(\hat{\mathbf{q}} \times \mathbf{N}(\mathbf{q})) \Big\} \\ (M - E_p)\mathbf{N}(\mathbf{p}) &= \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}-\mathbf{q}) \qquad\qquad (72) \\ &\times \Big\{ [c_p c_q + s_p s_q ((\hat{\mathbf{p}}\!\cdot\!\hat{\mathbf{q}}) - (\hat{\mathbf{p}} \times \hat{\mathbf{q}}) \times\; .\,))]\, \mathbf{N}(\mathbf{q}) \;+\; s_p s_q\, \hat{\mathbf{p}} \times \hat{\mathbf{q}}\, L(\mathbf{q}) \Big\} \end{aligned}$$

The reduced equations eqs.(71, 72) exibit the heavy–quark spin symmetry, *i.e.*, the pseudoscalar bound state is degenerate with the vector, the scalar with the axial vector [32]. This symmetry is a consequence of the fact that the dynamics becomes independent of the heavy quark spin in the limit $m_Q \to \infty$. We can demonstrate this explicitly for S–wave bound states ($J^P = 0^-, 1^-$). In this case, the equation for the L– and $\mathbf{N}$–component decouple, and one may verify that if eq.(71) possesses a pseudoscalar solution of the form $L(\mathbf{q}) = f(|\mathbf{q}|)$, eq.(72) admits a vector solution corresponding to the same orbital wave function multiplied by a constant polarization vector, $\mathbf{N}(\mathbf{p}) = \vec{S} f(|\mathbf{p}|)$, $\vec{S} = \text{const.}$. (Here, we suppose that the 3–dimensional potential is a function of $|\mathbf{p} - \mathbf{q}|$ only.) A similar degeneracy holds for arbitrary angular momentum and both parities, as is seen from the corresponding partial–wave equations, eqs.(A.9, A.10) in appendix A.1. Thus, the bound state equations eqs.(63, 64) naturally realize the heavy–quark spin symmetry in the limit $m_Q \to \infty$.

Let us briefly discuss the non–relativistic limit of this description of mesons. In this case we take in eq.(69) also $\nu_j(|\mathbf{p}|) \equiv 0$ and obtain $c_p^{\pm} \equiv 1, s_p^{\pm} \equiv 0$. In addition, we approximate both quark energies by their non-relativistic values, $E_n(|\mathbf{p}|) = m_n + |\mathbf{p}|^2/2m_n$, $n = i, j$. In this limit, eqs.(71, 72) reduce to the well-known non-relativistic potential model for heavy quarkonia [5].

We have seen that the bilocal effective theory with instantaneous interaction provides a unified description of both light and heavy flavors. It should thus be well suited

to investigate the spectrum and decays of heavy–light bound states. Our intention here is not to perform an exhaustive calculation of the meson spectrum based on the Salpeter equations, eqs.(63, 64), but rather to obtain a quantitative estimate of the effects of dynamical chiral symmetry breaking on the heavy–light meson spectrum and decay constants. Specifically, we wish to demonstrate the phenomenological importance of describing chiral symmetry breaking self–consistently through the Schwinger–Dyson equation, eq.(44), rather than by a constant mass for the light quark, eq.(50). To this end, we compare the masses and decay constants of the D– and B–mesons calculated using the momentum–dependent quark self energy from eq.(44) with those obtained using a constant light quark mass, eq.(50).

To describe the D– and B–mesons, we take as potential the sum of a Coulomb and a linear potential, eq.(16), as has been used in the analysis of charmonium, $c\bar{c}$ [5]. In our investigations here we use the same potential for D– and B–mesons and ignore the running of α_s. For the parameters we take the values $\sigma = 0.41\,\mathrm{GeV}$ and $\alpha_s = 0.39$, which we determined by fitting the masses of the $1S$– and $2S$–state of $c\bar{c}$ (J/ψ) as well as those of $b\bar{b}$ (Υ), using eqs.(63, 64) in the non-relativistic limit, $\nu_{ip} = \nu_{jp} \equiv 0, E_i(|\mathbf{p}|) = E_j(|\mathbf{p}|) = m_c + \frac{1}{2}|\mathbf{p}|^2/m_c$. In this fit the quark masses were obtained as $m_c = 1.39\,\mathrm{GeV}$ and $m_b = 4.79\,\mathrm{GeV}$.

The result of a calculation of the masses and decay constants of pseudoscalar D– and B–mesons and their first radially excited states with the above potential is shown in table 1. There, results are given obtained with both the dynamical self–energy for the light flavor and with a constant light quark mass. For the sake of comparison we have chosen the constant light quark mass equal to the dynamically generated quark mass, eq.(49), at $|\mathbf{p}| = 0$, which for this potential is $m(0) = 0.082\,\mathrm{GeV}$, *cf.* fig.3. We have also included values obtained with a typical light quark "constituent" mass, $m_q = 0.33\,\mathrm{GeV}$. As can be seen, the use of the constant quark mass instead of the dynamical self–energy leads to considerable changes in the meson masses and decay constants. The increase in the decay constant of the ground–state D– and B–meson can be explained by the sensitivity of $F_{D,B}$ to the higher–momentum part of the wave function, which is larger if a constant quark mass is used. This is seen also from fig.7, which shows the radial wave functions in both cases. Note further that the D– and B–meson masses calculated with the dynamical quark mass agree rather well with the experimental values, much better than the ones obtained with either $m_q =$ const.. (If m_q is taken to be significantly smaller than the values in table 1, the bound state mass even increases.) However, one should not overemphasize the quantitative agreement, since we did not choose parameters such as to fit the strength of chiral symmetry breaking in the light sector. More observables (f_π, $\langle\bar{q}q\rangle$) should be involved in order to optimize the potential before drawing conclusions based on absolute values. Nevertheless, the results of table 1 show clearly that a dynamical description of chiral symmetry breaking is important, and that noticeable effects due to the dynamical nature of the quark masses occur already in the meson mass spectrum and decay constants. Our results confirm the conclusions of Kaburagi *et al.*, who investigated the dependence of heavy–light meson properties on the light quark mass

in the framework of the Dirac equation with a scalar confining potential [36]. It should be stressed, however, that the main point in having a description which takes in to account the dynamical breaking of chiral symmetry is a qualitative one. The clearest manifestations of the role of dynamical chiral symmetry in heavy–light systems may be seen not in the masses and decay constants but in more complicated observables, *e.g.* processes involving emission of pions. Such processes can be described using the effective bilocal meson action, eq.(20).

Finally, we would like to comment on the ordering of the pseudoscalar meson decay constants. We find F_B to be slightly larger than F_D, for both the dynamical and constant light quark mass. This is in agreement with the calculation of Cea *et al.* [37], who use a relativistic potential model, but contrary to most non-relativistic results, *cf.* [38] and references therein. In [37] it is argued that relativisitc effects spoil the simple proportionality $F_H \propto M_H^{-1/2}$. If we treat in our approach the heavy quark as non-relativistic, *i.e.*, if we solve eqs.(63, 64) with $\nu_{1p} \equiv 0$ and $E_{1p} = m_Q + \frac{1}{2}|\mathbf{p}|^2/m_Q$, we find F_D to be somewhat larger than F_B for both the dynamical self–energy and the constant quark mass for the light flavor. Thus, the pattern is reversed if the relativistic kinematics is abandoned. Note that our spectra were calculated using the same potential for D– and B–mesons; larger differences in the decay constants may occur with a running coupling constant.

5 Summary and outlook

In this paper we have presented a bilocal effective model, which describes in a unified way both light and heavy mesons. Chiral symmetry is broken spontaneously by the interactions which bind the mesons. The important feature of our model is that the potential kernel, eq.(10), moves together with the bound state, which leads to a relativistically covariant description of bound states. The bilocal meson action obtained from this model, eq.(20), provides the script for deriving equations for bound states and for the calculation of matrix elements. The correct relativistic kinematics is essential in describing meson decays and more complicated processes like semileptonic decays [18, 23]. This covariant formulation, which constitutes a relativistic extension of the usual Coulomb gauge, should also be useful in the light quark sector, for example in describing the pion electromagnetic form factor [39] or processes like $\rho \to \pi\pi$.

The Salpeter equations for the meson wave functions can be simplified considerably by a Foldy–Wouthusen transformation, eq.(56). Of practical importance is the fact that this transformation does not lead to complications in the numerical solution of the bound state equations if the momentum–space Multhopp method is employed. The equations exhibit both chiral symmetry and heavy–quark spin symmetry in dependence on the chiral angle specifying the quark single–particle spectrum.

With a phenomenological potential used in the description of charmonium, good results for the masses of the pseudoscalar D– and B–mesons are obtained if spontaneous chiral symmetry breaking is taken into account. Heavy–light meson masses and decay

constants are seen to be sensitive to the momentum dependence of the light quark self–energy. This demonstrates the necessity of formualting chiral symmetry breaking self–consistently and justifies the effort of solving the Schwinger–Dyson equation in the light flavor sector.

Our aim here has been to set up the framework for describing heavy–light meson bound states in a bilocal effective theory. We have tried to demonstrate the viability of this scheme and to get a quantitative picture of the meson spectrum. Clearly, there is much room to improve and extend the results presented here. For example, an intermediate–range potential or transverse gluon exchange could be included in addition to the Coulomb and confinement potential in order to increase the strength of chiral symmetry breaking in the light sector [30, 17]. Furthermore, an interesting possibility would be to consider the effective meson action with the covariant instantaneous interaction in the heavy–quark limit and perform a long–wavelength expansion, as outlined in [22]. This would allow one to make quantitative predictions for the lagrangian of heavy quark effective theory in dependence on the interaction potential assumed at quark level. Moreover, the covariant instantaneous formulation could be generalized to the particle–particle sector of the quark theory, eq.(1), to describe also baryons as bound states.

Acknowledgement

This work was begun during the Heisenberg–Landau Workshop "Effective QCD at finite temperature" at the Joint Institute of Nuclear Research in Dubna, May 19 – 23, 1993. Yu. L. K. thanks the University at Tübingen, C. W. the University at Rostock and the Joint Institute in Dubna for their hospitality.

A The Salpeter equation

A.1 Partial wave decomposition

In this section we decompose the Schrödinger–type equations in the rest frame, eqs.(63, 64), into equations for bound states of given total angular momentum and parity (*cf.* also [35]). This is achieved by expanding the functions $L(\mathbf{p})$, $\mathbf{N}(\mathbf{p})$ in spherical harmonics of total angular momentum, J. For the vector component it is convenient to use instead of the usual vector spherical harmonics the combinations

$$\begin{aligned} \mathbf{Y}^1_{JM} &= \hat{\mathbf{p}} Y_{JM} &&= \alpha \mathbf{Y}_{JJ-1M} - \beta \mathbf{Y}_{JJ+1M}, \\ \mathbf{Y}^2_{JM} &= (J(J+1))^{-1/2}\, \nabla_{\hat{\mathbf{p}}} Y_{JM} &&= \beta \mathbf{Y}_{JJ-1M} + \alpha \mathbf{Y}_{JJ+1M}, \\ \mathbf{Y}^3_{JM} &= -i(J(J+1))^{-1/2}\, \hat{\mathbf{p}} \times \nabla_{\hat{\mathbf{p}}} Y_{JM} &&= \mathbf{Y}_{JJM}, \end{aligned} \tag{A.1}$$

with

$$\alpha \;=\; \sqrt{\frac{J}{2J+1}}, \qquad \beta \;=\; \sqrt{\frac{J+1}{2J+1}}, \qquad \alpha^2 + \beta^2 \;=\; 1. \tag{A.2}$$

The new functions $\mathbf{Y}^\lambda_{JM}$ have simple transformation properties under the operations $\hat{\mathbf{p}} \times \mathbf{Y}^\lambda_{JM}$ and $\hat{\mathbf{p}} \cdot \mathbf{Y}^\lambda_{JM}$. They are orthogonal and normalized as the usual vector spherical harmonics. We thus write the component functions of eq.(62) as[6]

$$L^{(k)}(\mathbf{p}) \;=\; \frac{\ell^{(k)}_J(p)}{p} Y_{JM}(\hat{\mathbf{p}}), \qquad \mathbf{N}^{(k)}(\mathbf{p}) \;=\; \sum_{\lambda=1}^{3} \frac{n^{(k)}_{\lambda J}(p)}{p} \mathbf{Y}^\lambda_{JM}(\hat{\mathbf{p}}) \qquad (k = 1, 2). \tag{A.3}$$

For the angular matrix element of the potential kernel we use the definition

$$\frac{pq}{(2\pi)^3} \int d\Omega_p \int d\Omega_q Y^*_{L'M'}(\hat{\mathbf{p}}) V(\mathbf{p} - \mathbf{q}) Y_{LM}(\hat{\mathbf{q}}) \;=\; v_L(p, q) \delta_{LL'} \delta_{MM'}. \tag{A.4}$$

Note that the matrix element is independent of M, and that $v_L(p, q)$ is a symmetric functions of the radial variables p, q. The general expressions for $v_L(p, q)$ corresponding to the power–like potentials used in eq.(16) are given in table 2. From the formula

$$\frac{pq}{(2\pi)^3} \int d\Omega_p \int d\Omega_q \mathbf{Y}^*_{J'L'M'}(\hat{\mathbf{p}}) V(\mathbf{p} - \mathbf{q}) \mathbf{Y}_{JLM}(\hat{\mathbf{q}}) \;=\; v_L(p, q)\, \delta_{JJ'} \delta_{LL'} \delta_{MM'} \tag{A.5}$$

$$(L \;=\; J, J \pm 1)$$

we obtain the matrix elements

$$\frac{pq}{(2\pi)^3} \int d\Omega_p \int d\Omega_q \mathbf{Y}^{\lambda *}_{JM}(\hat{\mathbf{p}}) V(\mathbf{p} - \mathbf{q}) \mathbf{Y}^\rho_{JM}(\hat{\mathbf{q}}) \;=\;$$

$$\begin{cases} \alpha^2 v_{J-1} + \beta^2 v_{J+1} \;\equiv\; \overline{v}_J(p, q) & (\lambda, \rho) = (1, 1) \\ \beta^2 v_{J-1} + \alpha^2 v_{J+1} \;\equiv\; \overline{\overline{v}}_J(p, q) & (\lambda, \rho) = (2, 2) \\ v_J & (\lambda, \rho) = (3, 3) \\ \alpha\beta(v_{J-1} - v_{J+1}) \;\equiv\; \widetilde{v}_J(p, q) & (\lambda, \rho) = (1, 2), (2, 1) \end{cases} \tag{A.6}$$

[6]In this section, p, q denote $|\mathbf{p}|, |\mathbf{q}|$.

All other combinations vanish.

With the above definitions it is straightforward to reduce eqs.(63, 64) to the following two systems of radial equations,

$$\begin{aligned}
M\ell_J^{\binom{2}{1}}(p) - E_p\ell_J^{\binom{1}{2}}(p) &= \int_0^\infty dq \left\{ [c_p^\mp c_q^\mp v_J + s_p^\mp s_q^\mp \overline{v}_J]\ell_J^{\binom{1}{2}}(q) + i s_p^\mp s_q^\pm \widetilde{v}_J n_{3J}^{\binom{2}{1}}(q) \right\} \\
M n_{3J}^{\binom{2}{1}}(p) - E_p n_{3J}^{\binom{1}{2}}(p) &= \int_0^\infty dq \left\{ [c_p^\mp c_q^\mp v_J + s_p^\mp s_q^\mp \overline{\overline{v}}_J] n_{3J}^{\binom{1}{2}}(q) - i s_p^\mp s_q^\pm \widetilde{v}_J \ell_J^{\binom{2}{1}}(q) \right\}
\end{aligned} \tag{A.7}$$

and

$$\begin{aligned}
M n_{1J}^{\binom{2}{1}}(p) - E_p n_{1J}^{\binom{1}{2}}(p) &= \int_0^\infty dq \left\{ [c_p^\pm c_q^\pm \overline{v}_J + s_p^\pm s_q^\pm v_J] n_{1J}^{\binom{1}{2}}(q) + c_p^\pm c_q^\mp \widetilde{v}_J n_{2J}^{\binom{1}{2}}(q) \right\} \\
M n_{2J}^{\binom{2}{1}}(p) - E_p n_{2J}^{\binom{1}{2}}(p) &= \int_0^\infty dq \left\{ [c_p^\mp c_q^\mp \overline{\overline{v}}_J + s_p^\mp s_q^\mp v_J] n_{2J}^{\binom{1}{2}}(q) + c_p^\mp c_q^\pm \widetilde{v}_J n_{1J}^{\binom{1}{2}}(q) \right\}
\end{aligned} \tag{A.8}$$

Here, $v_J \equiv v_J(p,q)$ *etc.*. The radial equations describe bound states of parity $(-)^{J+1}$ and $(-)^J$, respectively.

In the heavy–quark limit, with the Foldy–Wouthuysen factors given by eq.(69), $c_p^\pm \equiv c_p, s_p^\pm \equiv \pm s_p$, the systems of radial equations eqs.(A.7, A.8) simplify to

$$(M - E_p)\begin{pmatrix} \ell_J \\ n_{3J} \end{pmatrix}(p) = \int_0^\infty dq \, [c_p c_q \begin{pmatrix} v_J & \\ & v_J \end{pmatrix} + s_p s_q \begin{pmatrix} \overline{v}_J & -i\widetilde{v}_J \\ i\widetilde{v}_J & \overline{\overline{v}}_J \end{pmatrix}] \begin{pmatrix} \ell_J \\ n_{3J} \end{pmatrix}(q) \tag{A.9}$$

$$(M - E_p)\begin{pmatrix} n_{1J} \\ n_{2J} \end{pmatrix}(p) = \int_0^\infty dq \, [c_p c_q \begin{pmatrix} \overline{v}_J & \widetilde{v}_J \\ \widetilde{v}_J & \overline{\overline{v}}_J \end{pmatrix} + s_p s_q \begin{pmatrix} v_J & \\ & v_J \end{pmatrix}] \begin{pmatrix} n_{1J} \\ n_{2J} \end{pmatrix}(q) \tag{A.10}$$

Here, we have dropped the indices on the wave function components in accordance with eq.(70). These reduced equations can be diagonalized by introducing new radial wave functions,

$$\begin{aligned}
a_{1J} &= \alpha\ell_J - i\beta n_{3J}, & b_{1J} &= \alpha n_{1J} + \beta n_{2J}, \\
a_{2J} &= -\beta\ell_J - i\alpha n_{3J}, & b_{2J} &= -\beta n_{1J} + \alpha n_{2J}.
\end{aligned} \tag{A.11}$$

For the transverse vector components this is equivalent to using in eq.(A.3) instead of the $\mathbf{Y}_{JM}^{\lambda}$ the ordinary vector spherical harmonics, $\mathbf{Y}_{JLM}$, as basis functions. Upon this the system eqs.(A.9, A.10) becomes

$$(M - E_p)\begin{pmatrix} a_{1J} \\ a_{2J} \end{pmatrix}(p) = \int_0^\infty dq \, [c_p c_q \begin{pmatrix} v_J & \\ & v_J \end{pmatrix} + s_p s_q \begin{pmatrix} v_{J-1} & \\ & v_{J+1} \end{pmatrix}] \begin{pmatrix} a_{1J} \\ a_{2J} \end{pmatrix}(q)$$

$$(\mathrm{A.12})$$

$$(M-E_p)\begin{pmatrix} b_{1J} \\ b_{2J} \end{pmatrix}(p) \;=\; \int_0^\infty dq\,[c_p c_q \begin{pmatrix} v_{J-1} & \\ & v_{J+1} \end{pmatrix} + s_p s_q \begin{pmatrix} v_J & \\ & v_J \end{pmatrix}] \begin{pmatrix} b_{1J} \\ b_{2J} \end{pmatrix}(q) \tag{A.13}$$

The new system eqs.(A.12, A.13) exhibits the heavy–quark spin symmetry. For a given J the equations for $a_{1,J+1}$ and $a_{2,J}$ coincide with those for $b_{2,J}$ and $b_{1,J+1}$, respectively. This means that the bound states come in degenerate pairs with total angular momentum J and $J+1$, for both positive and negative parity. For example, in this limit the 0^- (pseudoscalar) meson is degenerate with the 1^- (vector), and the 0^+ (scalar) meson with the 1^+ (axial vector).

A.2 Numerical solution: the Multhopp method

The radial equations, eqs.(A.7, A.8), are in general singular integral equations with power–like or logarithmic singularities at $q = p$. A simple and powerful numerical technique to solve such equations directly in momentum space is the Multhopp method, which has been used frequently in the context of non-relativistic and relativistic constituent quark models [24]. Of crucial importance for this method is the fact that for potentials with known singularities the integral of the basis functions with the singular part of the potential kernel can be split off and performed analytically, so that only finite integrals need to be evaluated numerically. In the radial equations eqs.(A.7, A.8) a new feature compared to the quark model is the presence of the momentum–dependent Foldy–Wouthuysen factors, $c_q^\pm, s_q^\pm$, modifying the integration kernel. Here, we briefly show how the difficulties presented by these additional form factors can be circumvented and the singular part of the integrals be separated as usual. Thus, the Multhopp method can be used just as efficiently in the case of a non-trivial single–particle spectrum as in standard quark model calculations.

For simplicity, we consider eq.(A.7) for a $J^P = 0^-$ bound state; the generalization to the coupled equations for $J > 0$ is straightforward. In this case $\widetilde{v}_0 = 0$, $\overline{v}_0 = v_1$ and eq.(A.7) for $\ell^{(1,2)}(p) \equiv \ell_0^{(1,2)}(p)$ simplifies to

$$M\ell^{\binom{2}{1}}(p) - E_p \ell^{\binom{1}{2}}(p) \;=\; \int_0^\infty dq\,[c_p^\mp c_q^\mp v_0(p,q) + s_p^\mp s_q^\mp v_1(p,q)]\ell^{\binom{1}{2}}(q) \tag{A.14}$$

The Multhopp method consists in converting eq.(A.14) into a matrix equation in momentum space. The range of momenta $(0,\infty)$ is mapped onto the interval $(0,\pi)$ by the coordinate transformation

$$p \;=\; \lambda \tan \tfrac{1}{2}\theta, \qquad\qquad q \;=\; \lambda \tan \tfrac{1}{2}\chi. \tag{A.15}$$

where λ is a scale parameter with dimensions of momentum. One then chooses as basis functions the finite set $(2/\pi)^{1/2} \sin i\theta,\ i = 1,\ldots N$. The expansion of $\ell^{(1,2)}(\theta)$ is equivalent to interpolating $\ell^{(1,2)}(\theta)$ at a set of angles $\theta_k = k\pi/(N{+}1),\ k = 1,\ldots N$, the

so-called Multhopp angles. By making use of the orthogonality relation for the basis functions one obtains from eq.(A.14) a $2N \times 2N$–eigenvalue equation for $\ell^{(1,2)}(\theta_k)$,

$$\sum_{k=1}^{N} B_{jk}^{\binom{1}{2}} \ell^{\binom{1}{2}}(\theta_k) \quad = \quad M\ell^{\binom{2}{1}}(\theta_j), \tag{A.16}$$

with

$$B_{jk}^{\binom{1}{2}} \quad = \quad E(\theta_j)\delta_{jk} + \frac{2}{N+1}\sum_{i=1}^{N} \sin i\theta_k B^{\binom{1}{2}}(i,\theta_j), \tag{A.17}$$

$$B^{\binom{1}{2}}(i,\theta_j) \quad = \quad \int_0^\pi d\chi \frac{\lambda}{2\cos^2\frac{1}{2}\chi}[c_\theta^\mp c_\chi^\mp v_0(\theta,\chi) + s_\theta^\mp s_\chi^\mp v_1(\theta,\chi)]\sin i\chi \tag{A.18}$$

Here, $\ell^{(1,2)}(\theta), E(\theta), c_\theta^\mp, s_\theta^\mp, v_0(\theta,\chi), v_1(\theta,\chi)$ are related to the corresponding functions in eq.(A.14) by the transformation, eq.(A.15). The solution of eq.(A.16) directly yields the momentum–space wave function at a discrete set of points [24]. The calculations quoted in section 4 were performed with $N = 20\ldots30$.

The main input to eq.(A.16) are the Multhopp integrals of the potential kernel, eq.(A.18). In particular, the singularity of the integration kernel at $q = p$ is absorbed in the integral over the basis functions, eq.(A.18). We now show that by a simple rearrangement of terms the part of eq.(A.18) containing the singularity can be separated even in the presence of the Foldy–Wouthuysen factors. The Coulomb potential produces only an integrable logarithmic singularity in the radial equation, *cf.* table 2, and poses no further problems. For the linear potential the singularity is of the type

$$v_L(p,q)_{\,\mathrm{lin}} \quad = \quad \frac{1}{(p-q)^2} + \text{log. div. terms.} \tag{A.19}$$

Note that the leading singularity is independent of the angular momentum, L. Let us consider the contribution of the leading singularity, eq.(A.19), to the radial integral of eq.(A.14). As the leading singularities of $v_0(p,q)$ and $v_1(p,q)$ are identical, the singular part of the integral in eq.(A.14) is given by the principal value integral

$$\mathrm{P}\int_0^\infty \frac{dq}{(q-p)^2}\,[c_p^\mp c_q^\mp + s_p^\mp s_q^\mp]\ell^{\binom{1}{2}}(q). \tag{A.20}$$

Since the radial wave functions satisfy $\ell^{(1,2)}(0) = \ell^{(1,2)}(\infty) = 0$, eq.(A.20) can be integrated by parts, which gives

$$\mathrm{P}\int_0^\infty \frac{dq}{(q-p)}\,\frac{d}{dq}\Big\{[c_p^\mp c_q^\mp + s_p^\mp s_q^\mp]\ell^{\binom{1}{2}}(q)\Big\}. \tag{A.21}$$

This can be rewritten as

$$
\begin{aligned}
& \mathrm{P}\int_0^\infty \frac{dq}{(q-p)}\,\frac{d}{dq}\ell^{\binom{1}{2}}(q) && \text{(A.22)}\\
& + \int_0^\infty \frac{dq}{(q-p)}\Big\{[c_p^\mp c_q^\mp + s_p^\mp s_q^\mp - 1]\frac{d}{dq}\ell^{\binom{1}{2}}(q) + [s_p^\mp c_q^\mp - c_p^\mp s_q^\mp]\ell^{\binom{1}{2}}(q)\,\frac{d}{dq}(\nu_{1q}\mp\nu_{2q})\Big\}
\end{aligned}
$$

Of the integrals in eq.(A.22) only the first one is singular, which does not involve the Foldy–Wouthuysen angle in the integrand, while the second one is non-singular. Thus, the radial integrals involving the linear potential can be rearranged such that the Foldy–Wouthuysen factors, which are in general known only numerically, enter only in non-singular integrals. In other words, the Foldy–Wouthuysen transformation does not modify the leading singularity of the radial equation, as is to be expected. After the substitution, eq.(A.15), the Multhopp integral corresponding to the singular part of eq.(A.22) can be performed analytically as usual [24], because the Foldy–Wouthuysen factors do not enter. The remaining integrals of eq.(A.22), which involve the Foldy–Wouthuysen factors, lead to Multhopp integrals with at most logarithmic singularities, which can be calculated efficiently with standard Fourier transform routines.

The above argument can easily be extended to the full equations eqs.(A.7, A.8) for $J > 0$. In fact, it follows from eq.(A.19) and eq.(A.2) that the leading singularities of $\overline{v}_J(p,q)$ and $\overline{\overline{v}}_J(p,q)$, as defined in eq.(A.6), are equal to that of $v_J(p,q)$, and that the leading singularities cancel in $\widetilde{v}_J(p,q)$.

For a pure oscillator potential the entire Multhopp integral, eq.(A.18), can be evaluated analytically. In this case the Multhopp method provides an alternative to solving the differential equation for the radial wave functions state by state [21], as the solution of eq.(A.16) immediately gives the ground state and the excited state spectrum.

The scale parameter λ is chosen to be of the order of the r.m.s. 3–momentum of the bound state wave function. Eigenvalues and eigenfunctions should not be sensitive to this choice. We emphasize that this method is not a variational approach, *i.e.*, no variation is performed with respect to λ, in contrast to the procedure of [34, 35].

B Normalization and decay constants

In this appendix we evaluate the matrix elements needed for the normalization of the bound state amplitude and for the calculation of the pseudoscalar meson decay constants within the bilocal field approach. We shall derive explicit expressions in terms of the bound state wave functions in the rest frame.

For the normalization of the bound state amplitude we consider the matrix element of the free part of the quark loop, eq.(27), between on–shell bound states $H \sim (q_i\bar{q}_j)$,

$$
\begin{aligned}
\langle H(\mathcal{P}'_H)|W^{(2)}_{\rm eff}|H(\mathcal{P}_H)\rangle &= (2\pi)^4\delta(\mathcal{P}'_H-\mathcal{P}_H)(2\omega'_H 2\omega_H)^{-1/2}\mathcal{G}^{-1}(\mathcal{P}_H), && \text{(B.1)}\\
\mathcal{G}^{-1}(\mathcal{P}_H) &= -i\tfrac{1}{2}N_{\rm c}{\rm Tr}\,[\overline{\Gamma}(\mathcal{P}_H)G_1\Gamma(\mathcal{P}_H)G_2]. && \text{(B.2)}
\end{aligned}
$$

The trace implies integration over the loop momentum. We evaluate $\mathcal{G}^{-1}$ in the rest frame of the bound state, $\mathcal{P}_H = (M_H, \vec{0})$. Writing $\Gamma(\mathbf{q}|\mathcal{P}_H) \equiv \Gamma(\mathbf{q})$, $\overline{\Gamma}(\mathbf{q}|\mathcal{P}_H) = \Gamma(\mathbf{q}|-\mathcal{P}_H) \equiv \overline{\Gamma}(\mathbf{q})$, we have

$$\begin{aligned}
\mathcal{G}^{-1}(M_H, \vec{0}) &= -i\tfrac{1}{2}N_\mathrm{c} \int \frac{d^4q}{(2\pi)^4} \mathrm{tr}\left[\Gamma(\mathbf{q}) G_i(q - \tfrac{1}{2}\mathcal{P}_H)\overline{\Gamma}(-\mathbf{q}) G_j(q + \tfrac{1}{2}\mathcal{P}_H)\right] \qquad \text{(B.3)} \\
&= -\tfrac{1}{2}N_\mathrm{c} \int \frac{d^3q}{(2\pi)^3} \mathrm{tr}\left[\frac{\overline{\Gamma}(-\mathbf{q})\Lambda_{i+}(\gamma_0\Gamma(\mathbf{q})\gamma_0)\overline{\Lambda}_{j-}}{E_q - M_H} + \frac{\overline{\Gamma}(-\mathbf{q})\Lambda_{i-}(\gamma_0\Gamma(\mathbf{q})\gamma_0)\overline{\Lambda}_{j+}}{E_q + M_H}\right].
\end{aligned}$$

The numerators here can be rewritten as

$$\begin{aligned}
&\mathrm{tr}\,[\overline{\Gamma}(-\mathbf{q})\Lambda^i_\pm(\gamma_0\Gamma(\mathbf{q})\gamma_0)\overline{\Lambda}^j_\mp] = \\
&\quad -\mathrm{tr}\,[(S_j^{-1}(\mathbf{q})\overline{\Gamma}(-\mathbf{q})S_i^{-1}(\mathbf{q}))\ \overset{0}{\Lambda}_\pm\ (S_i^{-1}(\mathbf{q})\Gamma(\mathbf{q})S_j^{-1}(\mathbf{q}))\ \overset{0}{\Lambda}_\mp], \qquad \text{(B.4)}
\end{aligned}$$

where we have used the relation between the free and the rotated projectors, eq.(41). We now use the Salpeter equation, in the rest frame, eq.(60), and the corresponding equation for $\overline{\Gamma}(\mathbf{q})$ to express eq.(B.3) in terms of the "undressed" wave function in the rest frame, eq.(56),

$$\begin{aligned}
\mathcal{G}^{-1}(M_H, \vec{0}) &= \tfrac{1}{2}N_\mathrm{c} \int \frac{d^3q}{(2\pi)^3} \Big\{(E_q - M_H)\mathrm{tr}\,[\overset{0}{\Lambda}_-\ \overline{\overset{0}{\Psi}}(-\mathbf{q})\ \overset{0}{\Lambda}_+\ \Psi(\mathbf{q})] \qquad \text{(B.5)} \\
&\quad + (E_q + M_H)\mathrm{tr}\,[\overset{0}{\Lambda}_+\ \overline{\overset{0}{\Psi}}(-\mathbf{q})\ \overset{0}{\Lambda}_-\ \Psi(\mathbf{q})]\Big\}.
\end{aligned}$$

In a general frame, M_H is to be replaced by $\sqrt{\mathcal{P}_H^2}$, and the integration in eq.(B.5) goes over the transverse momentum, $q^\perp$. The bound state amplitudes are normalized by the requirement that the dispersion relation be the same as that of a free relativistic particle,

$$\mathcal{P}_\mu \frac{\partial}{\partial \mathcal{P}_\mu} \mathcal{G}^{-1}(\mathcal{P})|_{\mathcal{P}^2 = M_H^2} = M_H^2. \qquad \text{(B.6)}$$

Note that for a co-moving instantaneous interaction, eq.(10), the interaction part of the effective action, eq.(20), does not contribute to the normalization matrix element because K^η depends on $\mathcal{P}$ only through the unit boost vector, η, and $\mathcal{P}_\mu(\partial/\partial\mathcal{P}_\mu)\eta = 0$. From eq.(B.5), the normalization condition in the rest frame can thus be expressed as

$$1 = \frac{N_\mathrm{c}}{2M_H} \int \frac{d^3p}{(2\pi)^3} \Big\{\mathrm{tr}\,[\overset{0}{\Lambda}_-\ \overline{\overset{0}{\Psi}}(-\mathbf{q})\ \overset{0}{\Lambda}_+\overset{0}{\Psi}\ (\mathbf{q})] - \mathrm{tr}\,[\overset{0}{\Lambda}_+\ \overline{\overset{0}{\Psi}}(-\mathbf{q})\ \overset{0}{\Lambda}_-\overset{0}{\Psi}\ (\mathbf{q})].\Big\} \qquad \text{(B.7)}$$

In terms of the components of the wave function, eq.(62), this reads

$$1 = \frac{2N_\mathrm{c}}{M_H} \int \frac{d^3p}{(2\pi)^3} \left(L^{(1)*}L^{(2)} + L^{(2)*}L^{(1)} + \mathbf{N}^{(1)*}\!\cdot\!\mathbf{N}^{(2)} + \mathbf{N}^{(2)*}\!\cdot\!\mathbf{N}^{(2)}\right), \qquad \text{(B.8)}$$

where $L^{(1)*} \equiv L^{(1)*}(\mathbf{q}) = L^{(1)}(-\mathbf{q})$, *etc.*.

We now consider the pseudoscalar meson decay constants. The matrix element of eq.(32) for the decay of a meson $H \sim (q_i \bar{q}_j)$ into a leptonic pair is given by

$$\langle l\nu(\mathcal{P}_L)|W^{(2)}_{\rm semi}|H(\mathcal{P}_H)\rangle = (2\pi)^4 \delta^{(4)}(\mathcal{P}_H - \mathcal{P}_L)\frac{G_F}{\sqrt{2}}\langle l\nu|l_\mu|0\rangle \mathcal{F}^\mu(\mathcal{P}_H), \tag{B.9}$$

$$\mathcal{F}^\mu(\mathcal{P}_H) = -iN_{\rm c}\int \frac{d^4q}{(2\pi)^4}{\rm tr}_\gamma[O^\mu G_i(q - \tfrac{1}{2}\mathcal{P}_H)\overline{\Gamma}(q^\perp|\mathcal{P}_H)G_j(q + \tfrac{1}{2}\mathcal{P}_H)]. \tag{B.10}$$

By integrating over the parallel component of the loop momentum, this matrix element can be evaluated in terms of the wave function of the "moving" bound state, eq.(52),

$$\mathcal{F}^\mu(\mathcal{P}_H) = N_{\rm c}\int \frac{d^3q^\perp}{(2\pi)^3}{\rm tr}_\gamma[O^\mu \overline{\Psi}(q^\perp|\mathcal{P}_H)], \tag{B.11}$$

where $\overline{\Psi}(q^\perp|\mathcal{P}_H) \equiv \Psi(q^\perp| - \mathcal{P}_H)$. The decay constant is then read off by comparing eq.(B.9) with the general definition

$$\langle l\nu(\mathcal{P}_L)|W^{(2)}_{\rm semi}|H_{ij}(\mathcal{P}_H)\rangle = (2\pi)^4 \delta^4(\mathcal{P}_H - \mathcal{P}_L)\frac{G_F}{\sqrt{2}}F_{ij}\mathcal{P}^\mu_H\langle l\nu|l_\mu|0\rangle. \tag{B.12}$$

Note that for 3–dimensional interactions of the form eq.(10) in general different values for the timelike and spacelike part of the pion decay constant are obtained [21].

In this paper we only consider the decays of pseudoscalar mesons ($J^P = 0^-$) at rest, $\mathcal{P}_H = (M_H, \vec{0})$. In this case the wave function has the form

$$\overline{\overset{0}{\Psi}}(\mathbf{q}) = \left\{L^{(1)}(\mathbf{q}) - \gamma_0 L^{(2)}(\mathbf{q})\right\}\gamma^5, \tag{B.13}$$

as can be seen from the partial–wave expansion in appendix A. Inserting this expression with eq.(56) into eq.(B.11) and calculating the trace we obtain the decay constant of a pseudoscalar meson $H \sim (q_i \bar{q}_j)$ at rest,

$$F_H = \frac{4N_c}{M_H}\int \frac{d^3q}{(2\pi)^3}L^{(2)}(\mathbf{q})\cos(\nu_{iq} + \nu_{jq}). \tag{B.14}$$

Here, the wave functions in the rest frame $L^{(1,2)}(\mathbf{q})$ satisfy eqs.(63, 64) and are normalized according to eq.(B.8). In particular, in the isospin limit, $\nu_{iq} = \nu_{jq} = \frac{1}{2}(\frac{1}{2}\pi - \phi(|\mathbf{q}|))$, the decay constant becomes

$$F_H = \frac{4N_c}{M_H}\int \frac{d^3q}{(2\pi)^3}L^{(2)}(\mathbf{q})\sin\phi(|\mathbf{q}|). \tag{B.15}$$

For light mesons (π, K) it is customary to define the decay constant as $f_\pi = F_\pi/\sqrt{2}$, *etc.*.

References

[1] For a recent review see A. Ali in B Decays, ed. S. Stone, World Scientific, Singapore (1992)

[2] D. Ebert and M.K. Volkov, Z. Phys. **C 16** (1983) 205;
M.K. Volkov, Ann. Phys. **157** (1984) 282;
D. Ebert, Z. Phys. **C 28** (1985) 433

[3] D. Ebert and H. Reinhardt, Nucl. Phys. **B 271** (1986) 188 and Phys. Lett. **B 173** (1986) 453

[4] S. Klimt, M. Lutz, U. Vogl and W. Weise, Nucl. Phys. **A 516** (1990) 429

[5] E. Eichten *et al.*, Phys. Rev. **D 17** (1978) 3090; *ibid.* **D 21** (1980) 203

[6] For a recent review see S.N. Mukherjee *et al.*, Phys. Rep. **231** (1993) 201

[7] For recent reviews see
M.B. Wise, Lectures given at the CCAST Symposium on Particle Physics at the Fermi Scale, May – Jun. 1993, Caltech preprint CALT-68-1860 (1993);
H. Georgi, in Proceedings of the Theoretical Advanced Study Institute, eds. R.K. Ellis, C.T. Hill and J.D. Lykken, World Scientific, Singapore (1992) 589;
B. Grinstein, Ann. Rev. Nucl. Part. Sci. **42** (1992) 101

[8] E. Eichten and B. Hill, Phys. Lett **B 234** (1990) 511;
H. Georgi, Phys. Lett. **B 240** (1990) 447;
B. Grinstein, Nucl. Phys. **B 339** (1990) 253

[9] H. Kleinert, Phys. Lett. **B 62** (1976) 429 and in Understanding the Fundamental Constituents of Matter (Erice Lectures 1976), ed. A. Zichichi, Plenum, New York (1978)

[10] V.N. Pervushin and D. Ebert, Theor. Math. Phys. **36** (1979) 759;
D. Ebert, H. Reinhardt and V.N. Pervushin, Sov. J. Part. Nucl. **10** (1979) 444

[11] Yu.L. Kalinovsky *et al.*, Sov. J. Nucl. Phys. **49** (1989) 1059;
V.N. Pervushin, Nucl. Phys. **B 15** (Proc. Suppl.) (1990) 197

[12] V.N. Pervushin *et al.*, Fortschr. Phys. **38** (1990) 333;
Yu.L. Kalinovsky *et al.*, Few–Body Systems **10** (1991) 87

[13] Yu.L. Kalinovsky, L. Kaschluhn and V.N. Pervushin, Fortschr. Phys. **38** (1990) 353

[14] J.R. Finger and J.E. Mandula, Nucl. Phys. **B 199** (1982) 168

[15] A. Le Yaouanc, L. Oliver, O. Pène and J.–C. Raynal, Phys. Rev. **D 29** (1984) 1233

[16] S.L. Adler and A.C. Davis, Nucl. Phys. **B 244** (1984) 469

[17] R. Alkofer and P.A. Amundsen, Nucl. Phys. **B 306** (1988) 305

[18] F. Hussain *et al.*, Phys. Lett. **B 249** (1990) 295;
F. Hussain, J.G. Körner, M. Krämer and G. Thompson, Z. Phys. **C 51** (1991) 321;
F. Hussain *et al.*, Nucl. Phys. **B 370** (1992) 259;
S. Balk, J.G. Körner, G. Thompson and F. Hussain, Z. Phys. **C 59** (1993) 283

[19] H.–Y. Jin, C.–S. Huang and Y.–B. Dai, Z. Phys. **C 56** (1993) 707;
Y.–B. Dai, C.–S. Huang and H.–Y. Jin, Z. Phys. **C 60** (1993) 527

[20] Yu.L. Kalinovsky, L. Kaschluhn and V.N. Pervushin, Phys. Lett. **B 231** (1989) 288

[21] A. Le Yaouanc *et al.*, Phys. Rev. **D 31** (1985) 137

[22] M.A. Nowak and I. Zahed, Phys. Rev. **D 48** (1993) 356;
M.A. Nowak, M. Rho and I. Zahed, Phys. Rev. **D 48** (1993) 4370

[23] Yu.L. Kalinovsky and C. Weiss, in preparation

[24] S. Boukraa and J.–L. Basdevant, J. Math. Phys. **30** (1989) 1060

[25] R.T. Cahill, J. Praschifka and C.J. Burden, Aust. J. Phys. **42** (1989) 161

[26] J. Praschifka, C.D. Roberts and R.T. Cahill, Phys. Rev. **D 36** (1987) 209;
C.D. Roberts, R.T. Cahill and J. Praschifka, Ann. Phys. **188** (1988) 20

[27] H.J. Munczek and P. Jain, Phys. Rev. **D 46** (1992) 438;
P. Jain and H.J. Munczek, Phys. Rev. **D 48** (1993) 5403

[28] L. v. Smekal, P. A. Amundsen and R. Alkofer, Nucl. Phys. **A 529** (1991) 633

[29] V.N. Pervushin, Riv. Nuovo Cim. **8** Nr. 10 (1985) 1;
Nguyen Suan Han and V.N. Pervushin, Mod. Phys. Lett. **A 2** (1987) 367 and Fortschr. Phys. **37** (1989) 611

[30] M. Hirata, Phys. Rev. **D 39** (1989) 1425

[31] C.R. Münz, J. Resag, B.C. Metsch and H.R. Petry, Bonn University preprint TK–93–14 (1993)

[32] N. Isgur and M.B. Wise, Phys. Lett. **B 232** (1989) 113; *ibid.* **B 237** (1990) 527

[33] S.L. Adler and T. Piran, Rev. Mod. Phys. **56** (1984) 1

[34] J. Resag, C.R. Münz, B.C. Metsch and H.R. Petry, Bonn University preprint TK–93–13 (1993)

[35] J.–F. Lagaë, Phys. Rev. **D 45** (1992) 305, 317

[36] M. Kaburagi *et al.*, Z. Phys. **C 9** (1981) 213

[37] P. Cea, P. Colangelo, L. Cosmai and G. Nardulli, Phys. Lett. **B 206** (1988) 691

[38] The MARK III Collab.: J. Adler *et al.*, Phys. Rev. Lett. **60** (1988) 1375

[39] K. Langfeld, R. Alkofer and P.A. Amundsen, Z. Phys. **C 42** (1989) 159

Tables

	dynamical	$m_q = 0.082$	$m_q = 0.33$	exp.
m_D	2.03	2.33	2.34	1.87
F_D	0.16	0.21	0.25	≤ 0.29
$m_{D'}$	2.54	2.85	2.89	
$F_{D'}$	0.16	0.16	0.19	
m_B	5.38	5.64	5.65	5.28
F_B	0.18	0.24	0.29	
$m_{B'}$	5.83	6.11	6.14	
$F_{B'}$	0.21	0.23	0.26	

Table 1: The masses and decay constants of the 0^- D– and B–mesons and their first radially excited states for a linear plus Coulomb potential with $\sigma = 0.41\,\mathrm{GeV}$, $\alpha_s = 0.39$, $m_c = 1.39\,\mathrm{GeV}$ and $m_b = 4.79\,\mathrm{GeV}$. All energies in GeV. Masses and wave functions were determined from eq.(63), decay constants from eq.(B.14). Columns 1–3 refer to different descriptions of the light quark spectrum. 1: dynamical chiral symmetry breaking, eq.(44), with $m^0 = 1\,\mathrm{MeV}$, 2: constant light quark mass, eq.(50), $m_q = m(|\mathbf{q}| = 0) = 0.082\,\mathrm{GeV}$, 3: constant light quark mass, $m_q = 0.33\,\mathrm{GeV}$. The experimental bound on F_D in column 4 is from ref. [38].

$V(r)$	$V(\mathbf{p}-\mathbf{q})$	$v_L(p,q)$
$1/r$	$\dfrac{4\pi}{\vert\mathbf{p}-\mathbf{q}\vert^2}$	$\dfrac{1}{\pi}Q_L(x)$
r	$-\dfrac{8\pi}{\vert\mathbf{p}-\mathbf{q}\vert^4}+C\delta^{(3)}(\mathbf{p}-\mathbf{q})$	$\dfrac{1}{\pi}\dfrac{1}{pq}\dfrac{dQ_L}{dx}(x)$
r^2	$(2\pi)^3\nabla^2_{\mathbf{q}}\delta^{(3)}(\mathbf{p}-\mathbf{q})$	$\left(\dfrac{d^2}{dq^2}+\dfrac{L(L+1)}{q^2}\right)\delta(p-q)$

Table 2: The Fourier transforms and the angular matrix elements, eq.(A.4), corresponding to the power–like potentials used in eq.(16). The linear potential involves a contact term, $C = 8\pi\int\frac{d^3q}{(2\pi)^3}|\mathbf{q}|^{-4}$, which cancels the IR–divergence. Here, $Q_L(x)$ is the Legendre function of the second kind, with $x = (p^2+q^2)/2pq$.

Figure captions

Fig.1: The chiral angle, $\phi(|\mathbf{p}|)$, from the Schwinger–Dyson equation for a light flavor ($m^0 = 1$ MeV). Solid line: linear plus Coulomb potential ($\sigma = 0.41\,\mathrm{GeV}$, $\alpha_s = 0.39$), dotted line: pure oscillator potential ($V_0 = 0.247\,\mathrm{GeV}$). The dashed line shows the value for a constant heavy quark mass, $m_Q = m_c = 1.39\,\mathrm{GeV}$.

Fig.2: The quark energy, $E(|\mathbf{p}|)$, from the Schwinger–Dyson equation for a light flavor ($m^0 = 1$ MeV). The potentials are those of fig.1. The dashed line shows the energy for a constant heavy quark mass, $m_Q = m_c = 1.39\,\mathrm{GeV}$. The straight solid line indicates the asymptotic behavior, $E = |\mathbf{p}|$.

Fig.3: The dynamical quark mass, $m(|\mathbf{p}|)$ of eq.(49), for a light flavor ($m^0 = 1$ MeV). The potentials are those of fig.1.

Fig.4: The squared pion mass, m_π^2, in the chiral limit, as a function of the current quark mass, $m_u^0 = m_d^0 \equiv m^0$. The potentials are to those of fig.1. Solid line: linear plus Coulomb potential, dotted line: oscillator potential.

Fig.5: The decay constants of the pion, f_π, and its first radially excited state, $f_{\pi'}$, in the chiral limit. The potentials are those of fig.1. Solid lines: linear plus Coulomb potential, dotted lines: oscillator potential. Note the different scales for f_π and $f_{\pi'}$. Here, $f_\pi = F_\pi/\sqrt{2}$, $f_{\pi'} = F_{\pi'}/\sqrt{2}$.

Fig.6: The radial wave functions of the pion, $\ell^{(1,2)}(|\mathbf{p}|)$, for a linear plus Coulomb

potential ($\sigma = 0.41\,\mathrm{GeV}$, $\alpha_s = 0.39$) and $m_u^0 = m_d^0 = 1\,\mathrm{MeV}$. Solid lines: $\ell^{(1)}$, dotted lines: $\ell^{(2)}$. Shown are the ground and first excited state. The single-particle spectrum is determined by the Schwinger–Dyson equation.

Fig.7: The (1)–component of the radial wave function of the D–meson, $\ell^{(1)}(|\mathbf{p}|)$, for a linear plus Coulomb potential ($\sigma = 0.41\,\mathrm{GeV}$, $\alpha_s = 0.39$) and $m_c = 1.39\,\mathrm{GeV}$. Solid line: dynamical quark self–energy for the light flavor ($m^0 = 1\,\mathrm{MeV}$), dotted line: constant light quark mass, $m_q = 0.082\,\mathrm{GeV}$. We have not shown $\ell^{(2)}$; it is very close to $\ell^{(1)}$ due to the large mass of the charmed quark.